Abstract

The subgoal-based relaxation is grounded on the identification of regression-based necessary conditions for the satisfaction of sets of numeric subgoals. So far, it has been used to define novel heuristics to provide guidance in problems exhibiting a pronounced numeric structure. This work investigates how to further exploit this relaxation to better integrate it within the search. It does so introducing the notion of a multi-repetition relaxed plan (MRP). A MRP enables a novel relaxed plan based heuristic, subgoaling based helpful actions, and the extraction of what we call investigatory heuristics for easier yet significant problems. It can be proved that the procedure terminates in a finite number of steps. Indeed, best achievers are calculated in polynomial time, the backward procedure is a greedy process that only appends the already established achiever to the working relaxed plan. It does need to account for the fact that the same actions can occur multiple times, with different multiplicity.

Key Ideas:
- Distinguish propositional from numeric terms
- Approximate numeric achievements through a multi-step regression
- Solve recurrence equation interleaving propositional and numeric regression throughout best achievers
- Approximate counting with continuous relaxation (enables looking for optimal solutions using actions in isolation)

State of the Art, Issues and Research Question

- Previous work and issues
  - Solving numeric planning through heuristic search: devise heuristics from relaxed version of the problem. Known schemas:
    - Interval-Based Relaxation (IBR) (Metric-FF, Colin, LPG,...). Generality ok, but can be very coarse relaxation for easier yet significant problems.
    - Subgoaling-Based Relaxation and heuristics. It manages to capture some information sharing.
    - Obviously overcounting!
- Focus and Research Questions
  - Subgoaling principle to design novel heuristics and search guidance in Simple Numeric Planning Problems
  - How to better exploit and integrate subgoaling relaxation into search?
  - Results:
    - More informed Helpful Actions
    - Up-To-Jumping Actions

Background: Subgoaling-Based Relaxation for Simple Numeric Planning

Definition (Simple Numeric Planning)
Let $P$ be a numeric planning problem. A numeric condition $c \in C$ is simple iff $c$ is a linear arithmetic expression and all variables in $c$ are affected only by action effects that increase/decrease them by constant values. Problem $P$ is simple iff all numeric preconditions of its actions and goals are simple.

Definition (m-times-regressor)
Let $c := \sum_{k \in N} (m_k \cdot x_k)$ be $m_x \geq 0$ with $c \leq (c_1, c_2, \ldots, c_m) \in C$ be a simple numeric condition. The $m$-times regressor $c\langle m \rangle := \sum_{x_k \neq 0} (m_k \cdot x_k + x) \cdot x_k \geq 0$.

Definition (Possible Achiever)
We say that $a$ is a possible achiever of $c$ in a state $s$ if there exists an $m \in N$ such that

$$h_{\text{MAF}}(s, c) = \begin{cases} 0 & \text{if } s = c \\ \min_{a \in \text{actions}} (\lambda(a) + h_{\text{MAF}}(s, \text{pre}(a))) & \text{if } a \text{ is PC} \\ \min_{a \in \text{actions}} (\lambda(a) + h_{\text{MAF}}(s, \text{pre}(a))) & \text{if } a \text{ is SC} \\ \sum_{c \in C\setminus\{s\}} h_{\text{MAF}}(s, c) & \text{if } |c| > 1 \end{cases}$$

Relaxed Plan Computation and the $h_{\text{MAP}}$ Heuristic

Definition (From the estimate to an actual plan)

$$h(s) = \{ \} \quad \text{if } s = c$$
$$h(s) = (a, m, \text{pre}(a)) \cup h(s, \text{pre}(a)) \quad \text{if } |c| = 1$$
$$h(s) = \min_{a \in \text{actions}} (\lambda(a) + h_{\text{MAF}}(s, \text{pre}(a))) \quad \text{if } c \text{ is PC}$$
$$h(s) = \min_{a \in \text{actions}} (\lambda(a) + h_{\text{MAF}}(s, \text{pre}(a))) \quad \text{if } c \text{ is SC}$$

Sailing Example

- Goal: reach area A1 and A2, wind from the north
- $h_{\text{MAP}} = 22$. Sums up all the actions required to get both $y - x \geq 10 \land y - x \geq 20$.

Helpful Actions and Up-To-Jumping Actions

Definition (Subgoaling Helpful Actions)
An action $a$ is said to be helpful in a state $s$ w.r.t. relaxed plan iff $a$ is applicable in $s$ and $\exists h \\ \exists \pi, \exists \text{PC}s, \exists \text{SC}s, \exists \text{MRP}_\text{max}$.

Definition (Up-to-jumping Action, Syntax and Semantics)
An up-to-jumping action is a pair $(a, m)$ where $a \in A$ and $m \in N$. An up-to-jumping action $(a, m)$ is executable in a state $s$ if the precondition of $a$ is satisfied in $s$ (as a regular action); the state resulting form executing $(a, m)$ is obtained by repeating the execution of $a$ up to $m$ executions in states or a state where the preconditions of $a$ are not satisfied is generated by $m < \text{min}$ executions of $a$.

Experimental Results

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Future Work

- Explore use of up-to-jumping actions with optimal planning
- Investigate the use of redundant constraints in relaxed-plan heuristics
- Verify compatibility with non-simple numeric planning problems

Note: Further details and technical specifications are omitted for brevity. The full document contains comprehensive analysis, experimental results, and additional research questions.