Constrained Flight Planning under Weather Uncertainty

A constrained stochastic shortest path problem consists of:
- an SSP $S$
- a set of constraints $C$, where each constraint:
  - comes with a secondary cost function $c_i$
  - bounds the expected cost of this function by some constant
  - e.g., $E[c_i] \leq 30$ minutes

A solution for a C-SSP is a potentially stochastic policy which minimizes costs and satisfies constraints over expectation.

Constrained Stochastic Shortest Path Problem

- Compute a flight plan for a given aircraft mission which minimizes fuel consumption,
- underlying uncertain weather effects
- meets time and convection constraints

In other words: we want to solve a constrained stochastic shortest path problem (C-SSP)

A solution for an SSP is a deterministic policy (mapping from states to actions) which minimizes costs.

Stochastic Shortest Path Problem

A Stochastic Shortest Path Problem $S$ consists of:
- a set of states $S$
  - current position, speed, altitude
- a set of actions $A$
  - fly to waypoint, change altitude, change speed
- a cost function $C$
  - represents fuel consumption
- an initial state $s_0$ and a set of goal states $S^*$
- a probabilistic transition function $P(y_{s,a} | s)$
  - requires access to a weather forecast model

We use a black box model that computes state transitions $\pi$

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Column Generation

Column Generation – Solve the problem ignoring constraints

1. Solve the problem ignoring constraints
2. Evaluate constraints
3. Modify problem to improve the current solution
   - adaptation of the primary cost function

- We can use any SSP algorithm to solve this subproblem
- Computes a deterministic policy with associated costs

Column Generation – Evaluate constraints on policy

- If no constraint is violated and solution cannot be improved $\Rightarrow$ return solution
- Otherwise, modify current subproblem:
  - change problem such that $c$ can not be optimal $\Rightarrow$ original problem with shifted cost function
  - shifted costs explore different trade-offs between constraints and cost

Heuristic Decomposition

- Incorporate constraints in primary cost function via scalarisation
- Scalarisation: new cost function becomes a linear combination of primary and secondary cost functions
- Determine: remove uncertain weather outcomes and use average travel time and fuel consumption for transitions
- Decompose: the resulting problem in a 2D and a 3D problem
  - horizontal (2D) planning phase computes least cost path based on the earth surface
  - vertical (3D) planning phase assigns each node in the 2D plan an altitude and speed level

Empirical Evaluation

- Evaluate on real-world data set
- 3 short, medium, and long distance flights
- weather data from June 2018
- BADA aircraft performance model
- Time window constraints
- Convection constraints
- Focus on deterministic policies

Contribution

- A C-SSP formulation of constrained flight planning under uncertainty
- A new algorithm for C-SSPs based on Column Generation
- An alternative approach that decomposes the problem into a 2D search and a greedy choice of altitude and speed from a set of heuristic strategies
- We evaluate on real flight routes with real weather data

Stochastic and Deterministic Policies

- If required, we can select the best deterministic policy
- Deterministic policy is not guaranteed to satisfy constraints
- Finding an optimal deterministic policy is NP-complete

Alternative approach to Column Generation: Heuristic Decomposition based on Determination

Constrained Flight Planning under Weather Uncertainty

Common approach for constrained deterministic shortest path problems based on linear programming.

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2. Evaluate constraints
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Column Generation in Practice

Each policy corresponds to a column in the LP
- LP solver computes a solution to the LP:
  - solution is a convex combination of policies
  - i.e., a probability distribution over deterministic policies
  - guarantees minimum primary cost
  - respects constraints over expectation

Stochastic and Deterministic Policies

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Empirical Evaluation

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