Analyzing and Avoiding Pathological Behavior in Parallel Best-First Search

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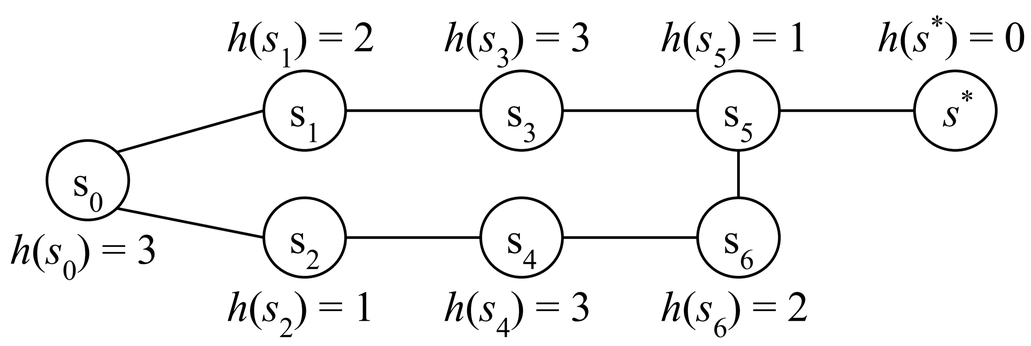
A Graph Search Problem with a Heuristic

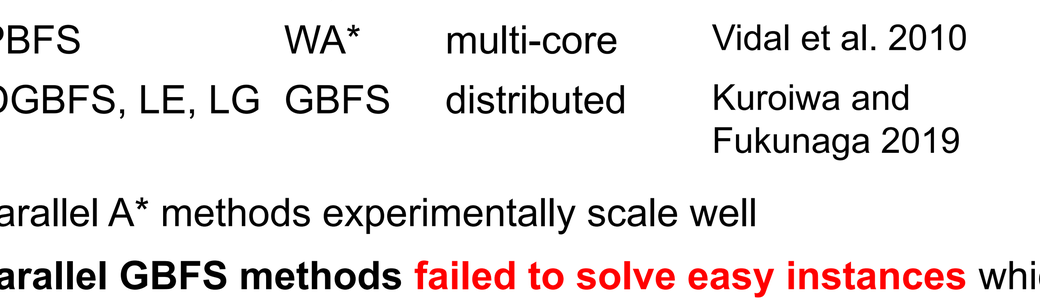


- A planning problem can be modeled as a graph search problem
- A solution is a path from s_0 to s^*
- The cost of a solution is the sum of costs of edges in the path
- The **heuristic** value $h(s)$ estimates the distance from s to s^*

Best-First Search (BFS) Algorithms

- Expands s with the minimum $f(s)$
- Break ties according to the **tie-breaking strategy**

GBFS expansion order



Method	Optimality	$f(s)$
A*, admissible heuristic	optimal	$f(s) = g(s) + h(s)$ $g(s)$: the path cost to s
Weighted A* (WA), admissible heuristic	w-suboptimal	$f(s) = g(s) + wh(s)$
Greedy Best-First Search (GBFS)	no guarantee	$f(s) = \mathrm{h}(s)$

Parallel Best-First Search Methods

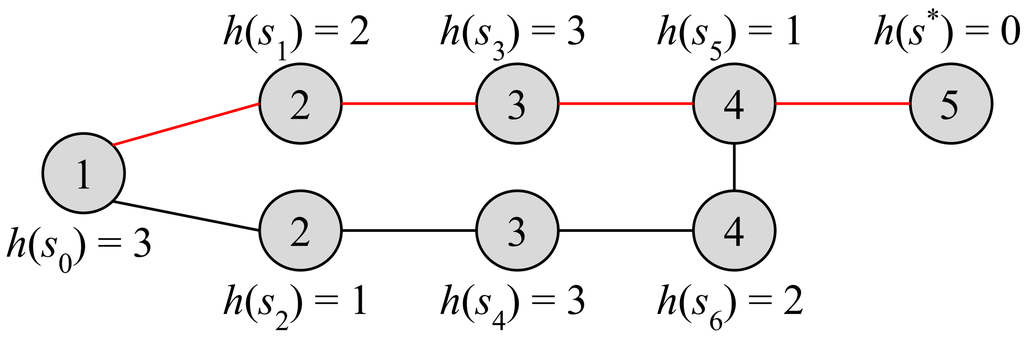
Method	BFS	Environment	Citation
HDA*	A*	distributed	Kishimoto et al. 2009
PBNF	A*, WA*	multi-core	Burns et al. 2010
KPBFS	WA*	multi-core	Vidal et al. 2010
HDGBFS, LE, LG	GBFS	distributed	Kuroiwa and Fukunaga 2019

- Parallel A* methods experimentally scale well
- **Parallel GBFS methods failed to solve easy instances** which GBFS solved within 1 sec. and 100 expansions
- **Parallel GBFS methods expanded >1000 times as many states** as GBFS in some instances

Theoretical Model for Parallel BFS

- Model parallel BFS as KBFS [Felner et al. 2003]
- Simultaneously expands k states

Parallel GBFS expansion order

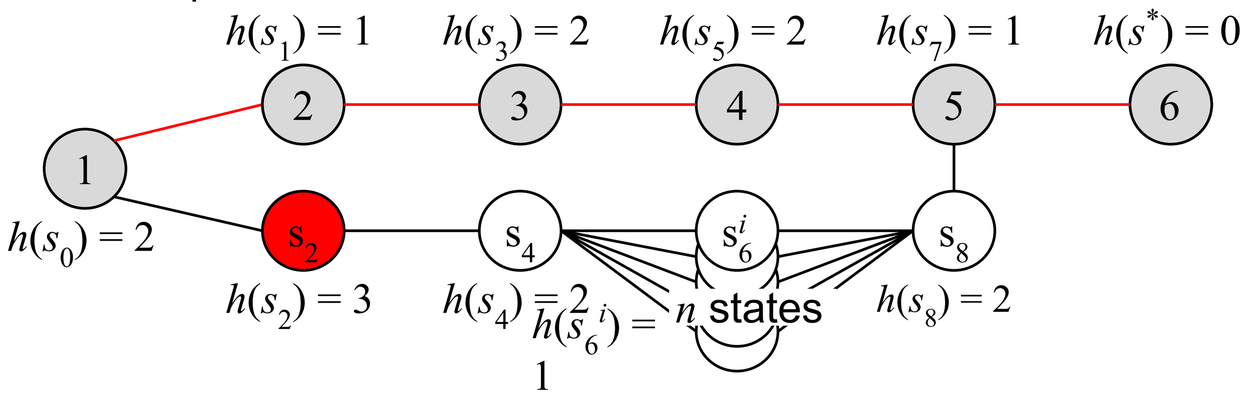


Theoretical Analysis of Parallel BFS

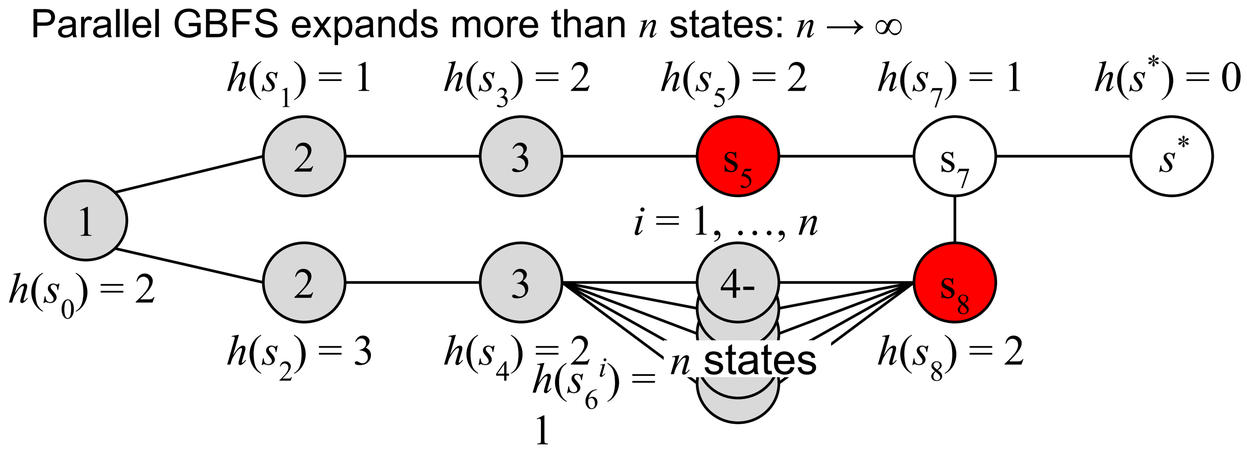
- A is **pathological** relative to B if A expands arbitrarily more states than B does given some graph and heuristic
- A is ***t*-bounded** relative to B if A expands no more than t times as many states as B does, for any graph and heuristic
- A is ***TB*-bounded** relative to B if A expands only states expanded by B with some tie-breaking strategy, for any graph and heuristic

Parallel GBFS is Pathological Relative to GBFS

GBFS expands 6 states



Parallel GBFS expands more than n states: $n \to \infty$



Parallel A* is *k*-Bounded Relative to A* with a Consistent Heuristic

- KA* (A* version of KBFS) with any tie-breaking strategy is k-bounded relative to A* with the worst-case tie-breaking strategy

Proof Sketch: A* expands s with min. $f(s) = g(s) + h(s)$ where $g(s)$ is the cost of the path from s_0 to s. A* expands each s with $f(s) < f^*$ (the cost of the optimal path). KA* expands s with $f(s) \leqq f^*$ at every k expansions because s' on an optimal path has $f(s') \leqq f^*$ by the consistency.

Bounded Parallel GBFS

P_{GBFS}/C: k-bounded and TB-bounded

- Execute k threads of independent GBFS with different tie-breaking strategies in parallel
- Use the **shared evaluation cache** of heuristic values

SPUHF: TB-bounded

- Expand s only if $h(s) \leqq h(parent(s))$ or no thread is expanding
- Execute independent parallel search using idling threads with the **shared evaluation cache**

Method	*t*-boundedness	*TB*-boundedness
HDA* (k processes), consistent heuristic	k-bounded	no
HDA*, inconsistent heuristic	pathological	no
KPBFS ($w > 1$)	pathological	no
KPBFS, HDGBFS, LE, LG	pathological	no
P_{GBFS}/C [New]	k-bounded	TB-bounded
SPUHF [New]	unknown	TB-bounded

Method	Coverage	# of solved instances unsolved by GBFS	# of unsolved instances solved by GBFS
LG	888	137	13
KPBFS	880	135	19
P_{GBFS}/C	**928**	164	**0**
SPUHF	864	115	15

55 domains from IPC-98-18. 5 min. time limit. 122 GiB memory limit.
16 threads / processes.