Certified Unsolvability for SAT Planning with Property Directed Reachability

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Certifying Algorithms

Algorithm emits certificate alongside its output, which is verified independently:

	SAT solvers	Planners
solvable	satisfying assignment	plan
unsolvable	DRAT proof	unsolvability certificate

Desired Properties

- sound & complete
- efficient generation (polynomial in planner runtime)
- efficient verification (polynomial in certificate size)
- generality

Unsolvability Certificates for Planning [E et al. 2018]

The certificate incrementally builds a knowledge base of proven statements:

- objects: state sets S_i (represented by propositional logic formulas φ_i)
- types of statements:
  - $S_1 \subseteq S_2$
  - S_1 dead (no state in S_1 can be part of a plan)
- basic statements:
  - state facts about concrete objects
  - need to be verified semantically
- derivation rules:
  - derive new knowledge from existing knowledge
  - universally true $\rightarrow$ only need to be verified syntactically

A Task is proven unsolvable if $\{I\}$ or G have been proven to be dead.

Basic Statements Examples

B1	$\bigcap S_1 \subseteq \bigcup S_2$	
B2	$(\bigcap S_1)[A] \subseteq \bigcup S_2$	$S[A] = \{s' \mid s \in S, s[a] = s' \text{ for some } a \in A\}$
B3	$[A](\bigcap S_1) \subseteq \bigcup S_2$	$[A]S = \{s \mid s' \in S, s[a] = s' \text{ for some } a \in A\}$

Derivation Rules Examples

Rules for showing deadness:

SD	S_1 dead, $S_2 \subseteq S_1$	$\rightarrow S_2$ dead
PG	$S_1[A] \subseteq S_1 \cup S_2$, S_2 dead, $S_1 \cap G$ dead	$\rightarrow S_1$ dead
RI	$[A]S_1 \subseteq S_1 \cup S_2$, S_2 dead, $\{I\} \subseteq \overline{S_1}$	$\rightarrow S_1$ dead
ED		$\rightarrow \emptyset$ dead

Rules from Set Theory:

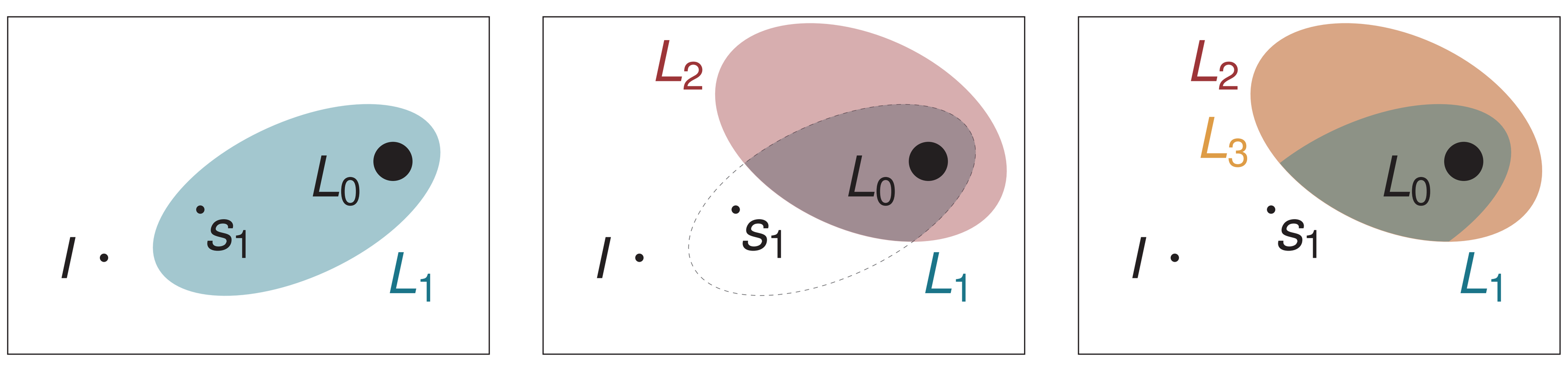
SI	$S_1 \subseteq S_2$, $S_1 \subseteq S_3$	$\rightarrow S_1 \subseteq S_2 \cap S_3$
ST	$S_1 \subseteq S_2$, $S_2 \subseteq S_3$	$\rightarrow S_1 \subseteq S_3$
UR		$\rightarrow S_1 \subseteq S_1 \cup S_2$

Property Directed Reachability [Suda 2014]

Property Directed Reachability (PDR) reasons about layers L_i which

- overapproximate states with distance $< i$ to goal,
- are iteratively refined, and
- are represented as CNF formulas, or Dual-Horn formulas for STRIPS tasks.

for $i = 0, \ldots$ **do**
  while $I \in L_i$ **do**
    if *exists path of length i from I to G* **then**
      return *found plan*
    else
      strengthen layers where path cannot be extended
    end
  end
  if $L_u = L_{u-1}$ *for some* $u < i$ **then**
    return *unsolvable*
  end
end

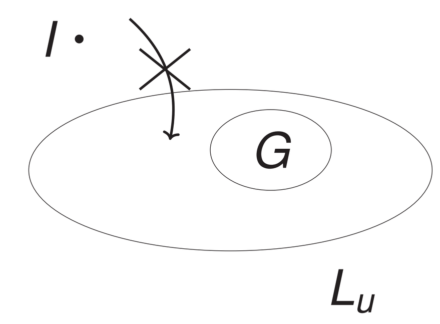


Certificate Structure

PDR's unsolvability argument:

- we cannot (backwards) reach new states from L_u
- L_u contains all goal states
- L_u does not contain the initial state

#	statement	justification
(1)	$[A]L_u \subseteq L_u$	basic statement
(2)	$\{I\} \subseteq \overline{L_u}$	basic statement
(3)	L_u is dead	from (1) and (2) with rule **RI**
(4)	$G \subseteq L_u$	basic statement
(5)	G is dead	from (3) and (4) with rule **SD**



Efficient Verification

Basic statements need to be verified semantically. If this can be done efficiently depends on the state set representation:

$$S_1 \subseteq S_2 \Leftrightarrow \varphi_1 \models \varphi_2$$

- efficient for BDDs, explicit enumeration, (Dual-)Horn and 2CNF formulas
- not efficient for CNF formulas

Basic Statements for CNF

Planner calls SAT solver, which is a certifying algorithm.
$\rightarrow$ Integrate UNSAT certificates into proof

	statement	required UNSAT certificate(s)
C1a	$S_1 \subseteq S_2$	$\varphi_1 \wedge \neg\gamma$ for each γ in φ_2
C1b	$S_1 \subseteq \overline{S_2}$	$\varphi_1 \wedge \varphi_2$
C2a	$S_1[A] \subseteq S_2$	$\varphi_1 \wedge T_A \wedge \neg\gamma'$ for each γ in φ_2
C2b	$S_1[A] \subseteq \overline{S_2}$	$\varphi_1 \wedge T_A \wedge \varphi_2'$
C3a	$[A]S_1 \subseteq S_2$	$\varphi_1' \wedge T_A \wedge \neg\gamma$ for each γ in φ_2
C3b	$[A]S_1 \subseteq \overline{S_2}$	$\varphi_1' \wedge T_A \wedge \varphi_2'$

- state sets S_i represented by CNF formulas $\varphi_i = \bigwedge \gamma_i$
- transition formula T_A encodes pairs of states (s, s') with $s[a] = s'$ for $a \in A$

Modified Certificate for PDR with SAT

The SAT calls performed by PDR don't match the required certificates.
$\rightarrow$ modify basic statements and use additional derivation rules:

#	statement	justification
(1a)	$[A]L_u \subseteq states(\gamma)$ for all γ in φ_{L_u}	SAT certificates provided by planner
(1b)	$[A]L_u \subseteq L_u$	from (1a) with rule **SI**
(2)	$\{I\} \subseteq \overline{L_u}$	build UNSAT certificate by hand*
(3)	L_u is dead	from (1b) and (2) with rule **RI**
(4a)	$G \subseteq states(\gamma)$ for all γ in φ_{L_u}	build UNSAT certificates by hand*
(4b)	$G \subseteq L_u$	from (4a) with rule **SI**
(5)	G is dead	from (3) and (4b) with rule **SD**

*formula can be proven unsolvable solely by unit propagation

Experimental Evaluation (PDR without SAT)

	base	certifying	verifier
PDR	388	384	382
FD-$h^{\mathrm{M\&S}}$	224	197	178
FD-$h^{\max}$	203	156	140
DFS-CL	394	386	385