On the Computational Complexity of Multi-Agent Pathfinding on Directed Graphs

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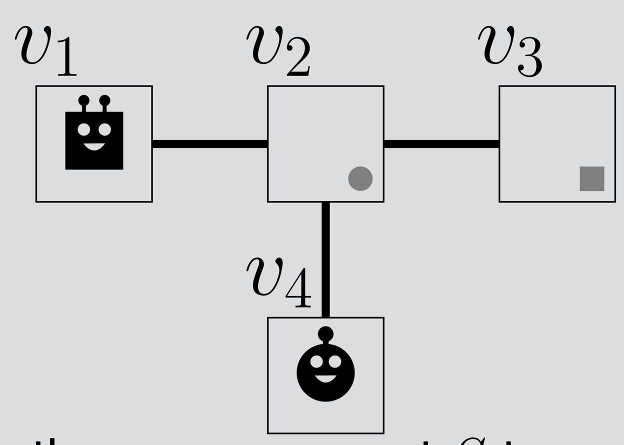
Multi-Agent Pathfinding

- **Given**: A set of agents A, an undirected, simple graph $G = (V, E)$, an initial state modelled by an injective function $\alpha_0 : A \to V$ and a goal state modelled by another injective function $\alpha_* : A \to V$.
- **Question**: Can α_0 be transformed into α_* by movements of single agents without collisions?
- **Example**:



- Find a plan to move the square agent S to v_3 and the circle agent C to v_2!

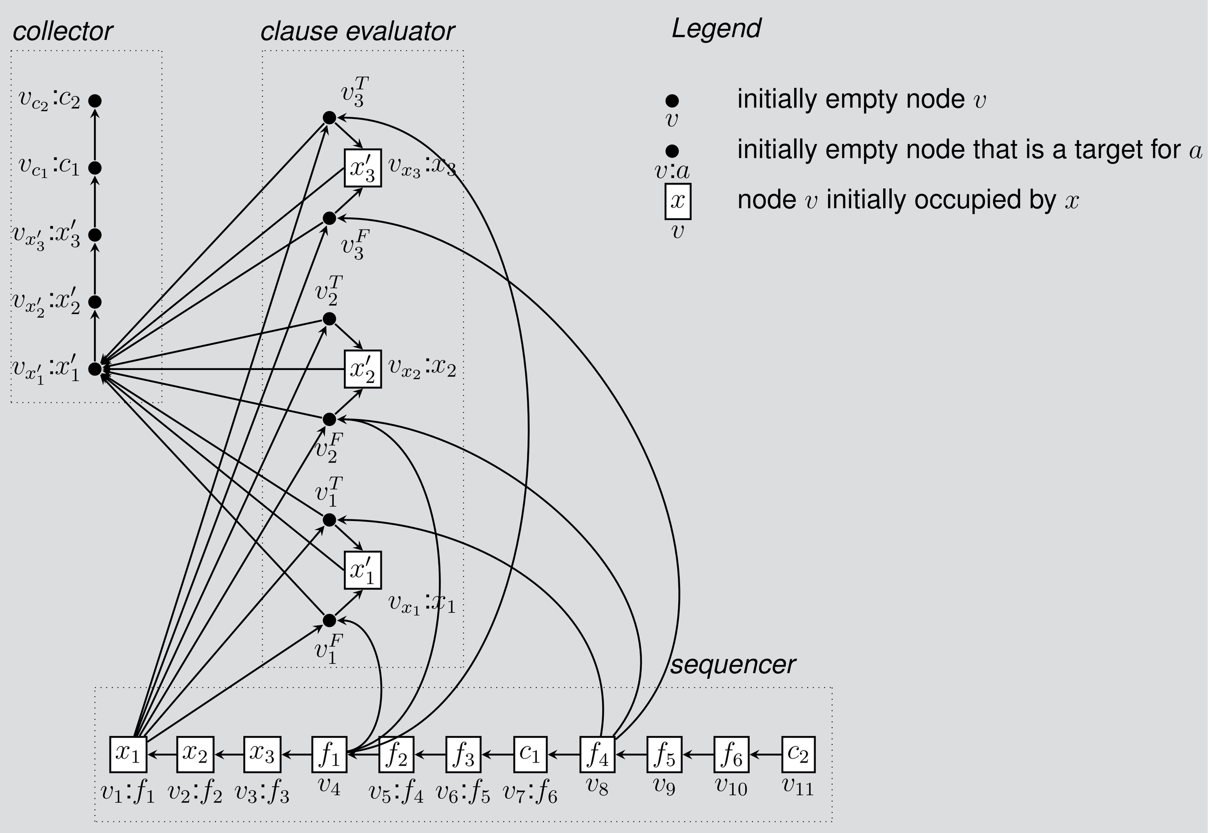
Known complexity results and open problems

- Deciding MAPF plan existence can be solved in $O(n^3)$ time and the plan length can be bounded by $O(n^3)$ movement actions [2].
- Finding a shortest plan is NP-complete [3].
- A number of variations of the problem (e.g. parallel movements) have been studied and optimization variants with parallel moves have been shown to be NP-complete [4, 5].
- Plan existence on strongly bi-connected directed graphs is polynomial [1].
- **Open problem since 1984:** What is the computational complexity of MAPF on general directed graphs (diMAPF)?

Main result: A lower bound

Theorem *diMAPF solvability is NP-hard.*

Proof: Reduction from 3SAT. Example reduction for $(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3)$:



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Two upper bounds

Theorem *diMAPF on DAGs is in NP.*

Proof: In each plan, an agent can visit each node only once. Hence, a plan can have only $O(n^2)$ steps, i.e. one can guess and check a solution in polynomial time. ■

Theorem *diMAPF on general directed graphs is in PSPACE.*

Proof: diMAPF can be easily reduced to propositional STRIPS, which is in PSPACE. ■

A conditional upper bound for the general case

Hypothesis *The solution length for diMAPF on strongly connected digraphs is polynomial.*

This hypothesis appears to be plausible. It is true for the special case of strongly bi-connected digraphs with at least two empty nodes.

Theorem *diMAPF is NP-complete, provided the short solution hypothesis is true.*

Proof: Each agent can only enter a strongly connected component once, and leave it once. So if there are short solutions for all strongly connected components, there will be one for the overall graph. ■

Summary

- Identified problem that is open since more than 35 years (but did anybody notice that it was open?)
- Demonstrated that a Kornhauser-style algorithm for directed graphs is impossible.
- Results generalize to variations with parallel moves.
- Open problem: Is the short solution hypothesis true?

References

[1] A. Botea, D. Bonusi, and P. Surynek. Solving multi-agent path finding on strongly biconnected digraphs. *Journal of Artificial Intelligence Research*, 62:273–314, 2018.

[2] D. Kornhauser, G. L. Miller, and P. G. Spirakis. Coordinating pebble motion on graphs, the diameter of permutation groups, and applications. In *25th Annual Symposium on Foundations of Computer Science (FOCS-84)*, pages 241–250, 1984.

[3] D. Ratner and M. K. Warmuth. Finding a shortest solution for the N $\times$ N extension of the 15-puzzle is intractable. In *Proceedings of the 5th National Conference on Artificial Intelligence (AAAI-86)*, pages 168–172, 1986.

[4] P. Surynek. An optimization variant of multi-robot path planning is intractable. In *Proceedings of the Twenty-Fourth Conference on Artificial Intelligence (AAAI-10)*, 2010.

[5] J. Yu and S. M. LaValle. Structure and intractability of optimal multi-robot path planning on graphs. In *Proceedings of the Twenty-Seventh Conference on Artificial Intelligence (AAAI-13)*, 2013.