



# Dynamic Controllability and (J,K)-Resiliency of Generalized Constraint Networks with Uncertainty

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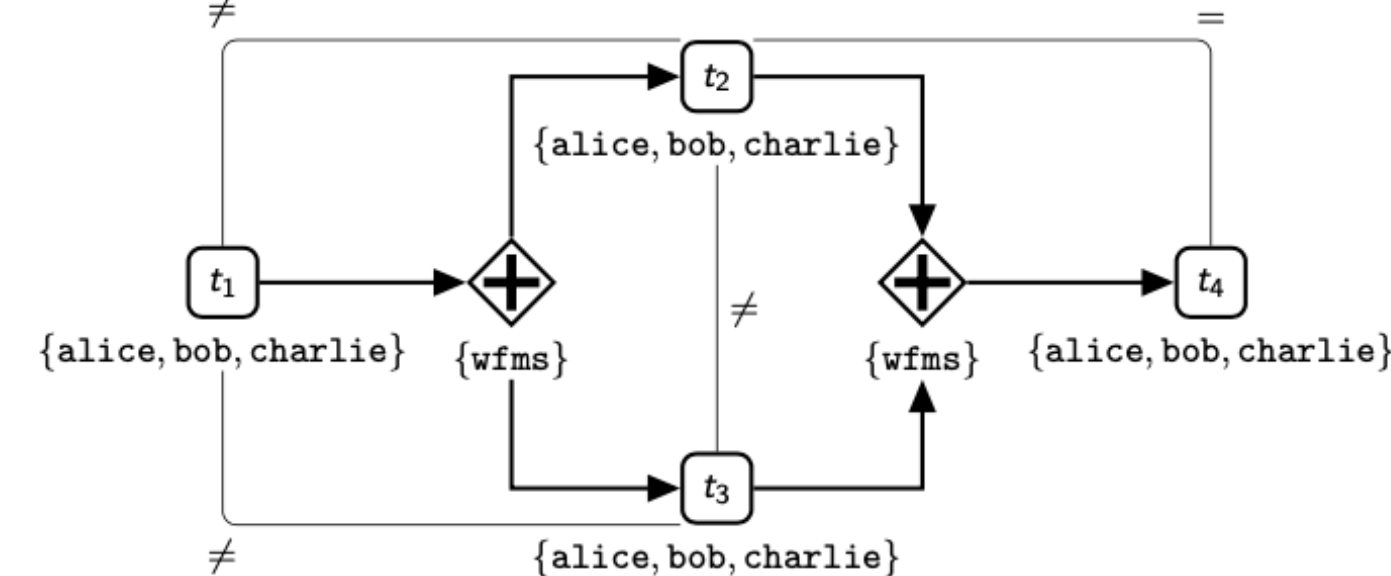
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## Aims of the Work

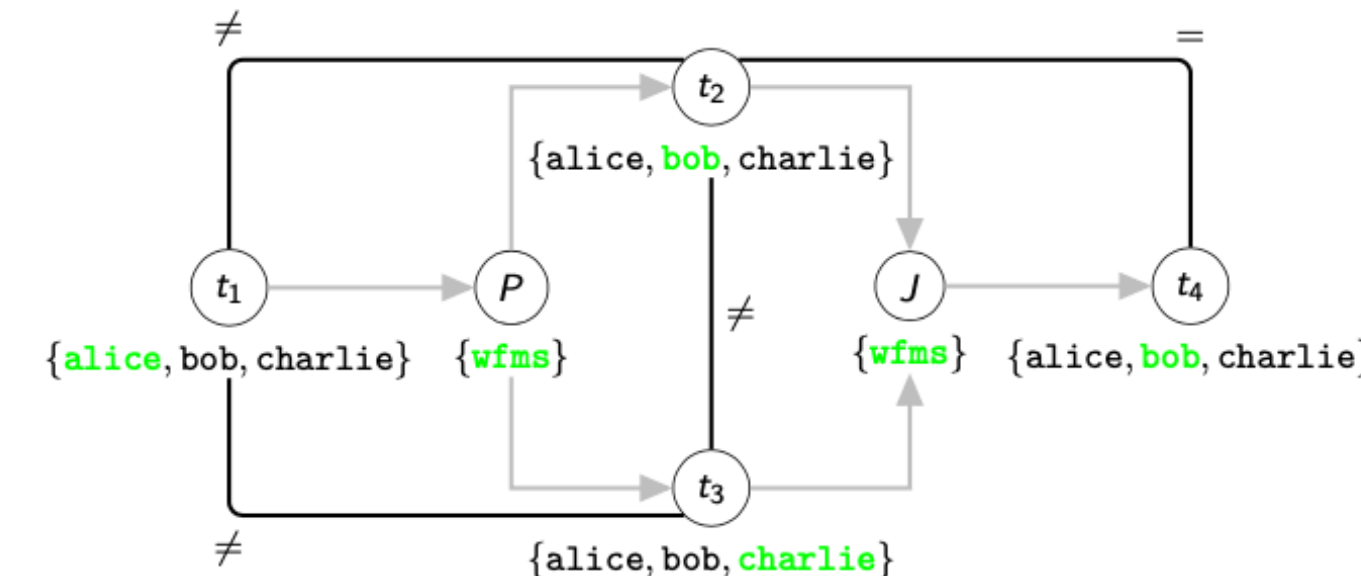
1. Handling resource allocation under uncertainty in a qualitative time domain.
2. Exploring resiliency (=sudden absence of resources) on top of dynamic controllability.
3. Estimating complexity of such a kind of planning.

## Constraint Networks for Resource Allocation

### Process with Resources



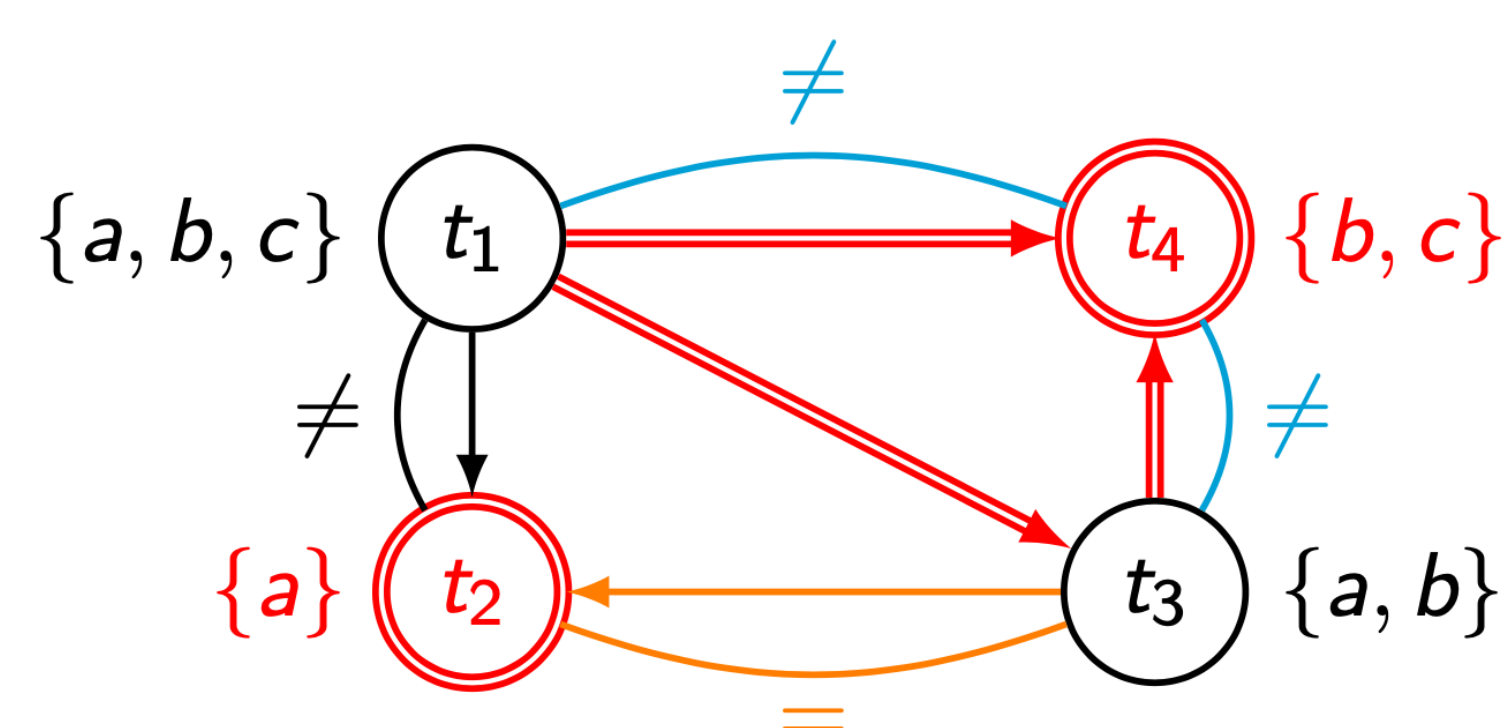
### Partially ordered CN



#### How to get a plan:

- 1) Compute a topological sort (polynomial-time step)
- 2) Solve the constraint network (NP-hard step)

## Generalized CNs with Uncertainty



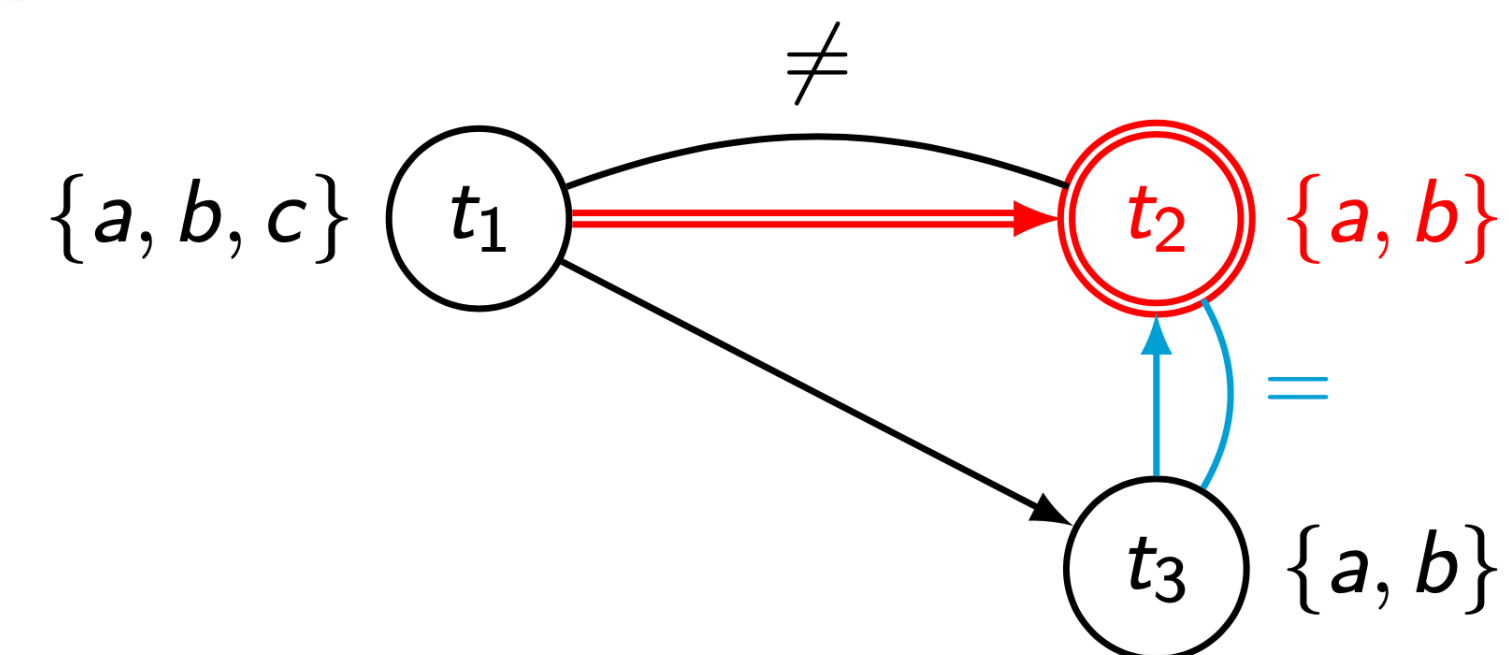
#### In words:

- 1) Variables with controllable and uncontrollable value assignment (double circles)
- 2) Variables with uncontrollable picking and (acyclic) activation constraints (double arrows)
- 3) Constraints language: arbitrary boolean formula over relational and partial order constraints.
- 4) Allow for defining problems as 2-player games Controller-Nature.
- 5) Qualitative time approach.

#### What we do:

- we execute (=pick then assign values) to variables one at a time.
- $t_2$  and  $t_4$  have uncontrollable value assignments.
- $t_3$  and  $t_4$  have uncontrollable pickings (=once active they can be executed "anytime").
- $(t_1 < t_2) \wedge (t_1 \neq t_2) \wedge (t_1 \neq t_4 \vee t_3 \neq t_4) \wedge (t_3 < t_2 \vee t_2 = t_3)$ .

## Dynamic Controllability



### Dynamically Controllable.

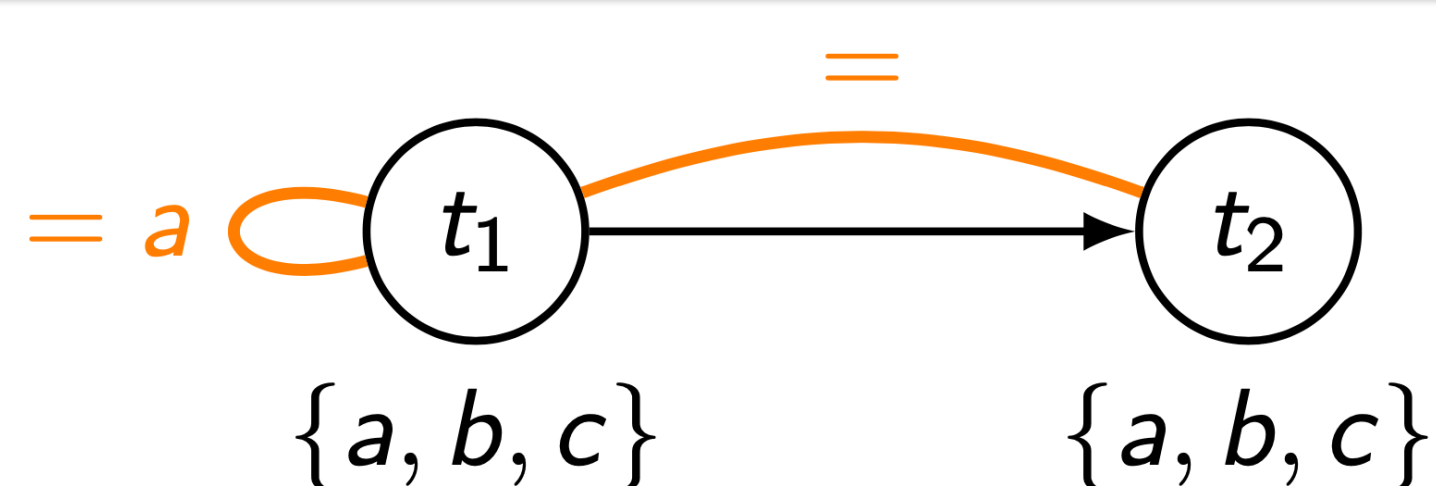
#### In words:

- 1) Round-based Game between Controller and Nature.
- 2) At every round a variable is *first* picked, then assigned a value.
- 3) Both Controller and Nature can pick and/or assign (=4 possible cases).
- 4) Nature has priority over Controller when picking.
- 5) Dynamic controllability = Winning strategy for Controller.
- 6) Uncontrollability = Winning strategy for Nature.

#### Controller's Winning Strategy:

- 1) Controller picks  $t_1$  and assigns  $c$  to it ( $t_2$  is now ready for picking)
- 2) If Nature doesn't pick  $t_2$ ,
  - 2.1) Controller picks  $t_3$  and assigns any value to it.
  - 2.2) Nature picks  $t_2$  and assigns a value to it
- 3) If Nature picks  $t_2$ , she also assigns a value to it.
- 4) Controller picks  $t_3$  and assigns to it the same value assigned to  $t_2$ .

## (J,K)-Decremental Resiliency



### Not (2,2)-decrementally resilient.

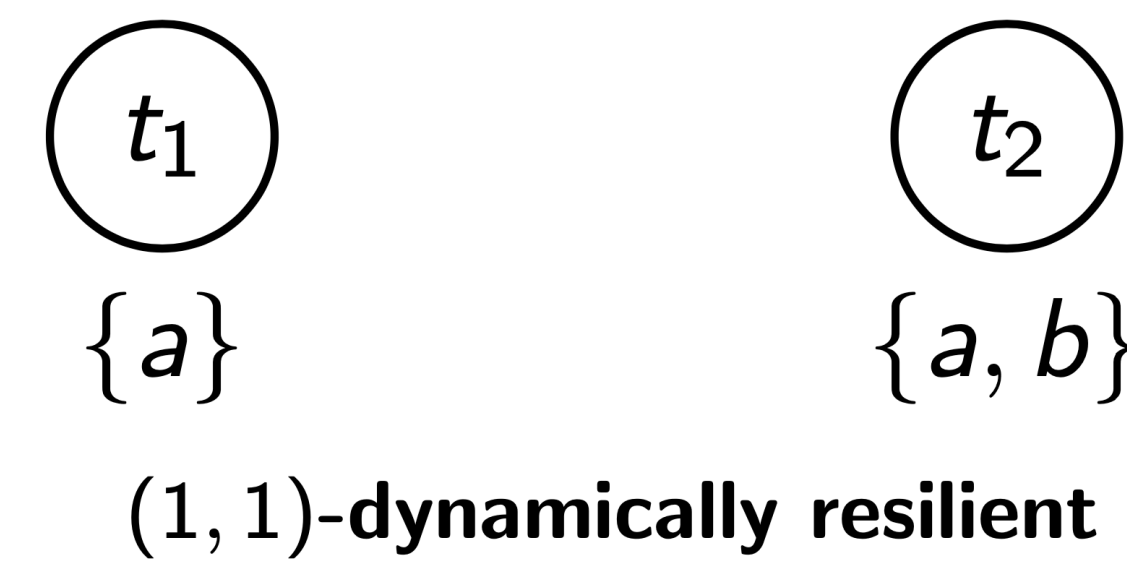
#### Nature's Winning Strategy:

- 1) Nature removes  $a$  at Round 1
- 2) Controller picks  $t_1$  and assigns  $b$  or  $c$  to it at Round 1
- 3) Nature removes the value that Controller assigned to  $t_2$  at Round 2.
- 4) Controller cannot assign the same value to  $t_2$  at Round 2 (i.e., loses the game).

#### In words:

- 1) On top of dynamic controllability.
- 2)  $J$  and  $K$  are natural numbers
- 3) For maximum  $J$  rounds Nature *strikes* by  $r$  removing *overall* up to  $K$  values
- 4) After that, Controller and Nature pick and assign values to variables (among those remained).
- 5) We still look for a winning strategy for Controller.
- 6)  $J$  depends on  $K$  (i.e.,  $J \leq K$ )

## (J,K)-Dynamic Resiliency



#### In words:

- 1) Like decremental, but with value re-entering at the end of round.
- 2) For maximum  $J$  times, at the start of every round, Nature strikes by removing up to  $K$  values.
- 3) After that, Controller and Nature pick and assign values to variables (among those remained).
- 4) Before the next round begins, all removed values become available again.
- 5)  $J$  is independent from  $K$ .

#### Controller's Winning Strategy:

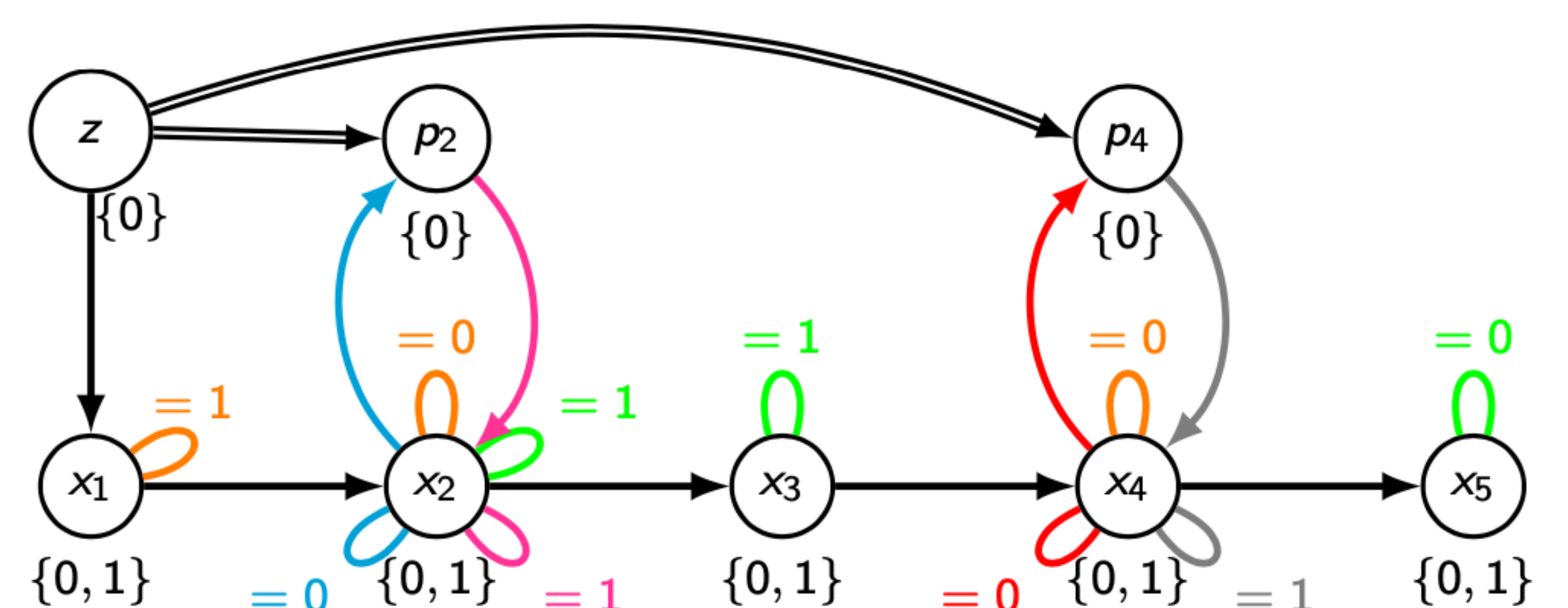
- 1) If Nature removes  $a$  at round 1,
  - 1.1) Controller picks  $t_2$  and assigns  $b$  to it.
  - 1.2) Controller picks  $t_1$  and assigns  $a$  to it
- 2) If Nature doesn't remove  $a$  at Round 1.
  - 2.1) Controller picks  $t_1$  and assigns  $a$  to it
  - 2.2) Controller picks  $t_2$  and assigns any value to it among those available.

## PSPACE-completeness

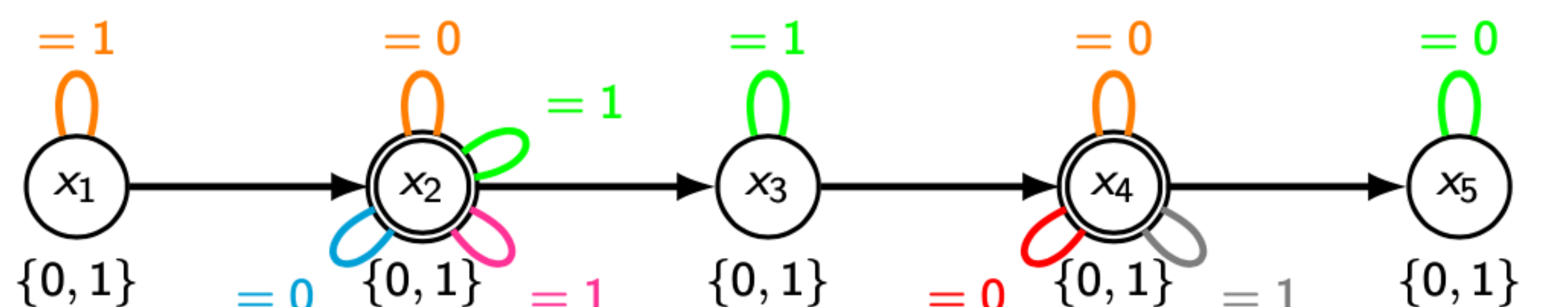
### Dynamic Controllability is PSPACE-complete

$$\exists x_1 \forall x_2 \exists x_3 \forall x_4 \exists x_5 (x_1 \vee \neg x_2 \vee \neg x_4) \wedge (x_2 \vee x_3 \vee \neg x_5)$$

## Uncontrollable pickings only



## Uncontrollable variable assignments only



- **Hardness:** reduction from true quantified boolean formula
- **Membership:** AND/OR search tree with depth bounded by a polynomial in the number of variables.

(J,K)-Decremental and (J,K)-Dynamic Resiliency are PSPACE-complete as well.

## Future Work

### Development of strategy synthesis algorithms

## References

Matteo Zavatteri, Romeo Rizzi, and Tiziano Villa. Dynamic Controllability and (J, K)- Resiliency in Generalized Constraint Networks with Uncertainty. In Proceedings of the Thirtieth International Conference on Automated Planning and Scheduling, ICAPS 2020, pages 314–322. AAAI Press, 2020.