1. Problem Statement

Environment – a graph (grid with 2\(^d\) connectedness or a roadmap)

Agent – an open disk of radii \(r\) that is allowed to:
- move with constant speed
- stay and wait any arbitrary amount of time
- rotate in place

Dynamic obstacles – \(N\) disks of radii \(r\) that move inside the environment along the known trajectories

Trajectory – a sequence of actions:
\[ \pi = (\theta_1, \theta_2, ..., \theta_n) \]

Task – having the given graph, a set of dynamic obstacles, start and goal agent’s locations, find a trajectory that avoids static and dynamic obstacles.

Solution quality is determined by the cost(dur) of the agent’s trajectory.

Cost – sum of actions’ durations:
\[ c(\pi) = \sum_{i=1}^{n} d ur(a_i) \]

Optimal Solution:
\[ \pi^* \text{ Plan with the minimal cost} \]

Bounded-suboptimal solution:
\[ c(\pi^*) \leq c(\pi^a), w \geq 1.0 \]

2. Safe Interval Path Planning


- **SIPP** is complete and optimal
- Each vertex has a corresponding safe intervals computed with respect to the trajectories of dynamic obstacles
- **SIPP state** – (graph vertex, safe interval) \( n = (v, [t, t]) \)
- \( g(n) \) – the earliest possible arrival time to \( n \)
- \( h(n) \) – estimate of the plan cost to the goal
  - Admissible
  - Consistent

When SIPP expands a node \( n = (v, [t, t]) \), then \( g(n) \) is the earliest possible arrival time to \( v \) in \( [t, t] \).

3. Bounded Suboptimal SIPP

**Weighted SIPP with re-expansions (WSIPP,\(_w\))**
- \( h\)-values are multiplied by factor \( w \) (like in WA*)
- Each state can be re-expanded multiple times
- Without allowed re-expansions loses the property of completeness (see example below)

**Weighted SIPP with Duplicate States (WSIPP,\(_d\))**
- Suggested by authors of SIPP in [Narayanan et. al, 2012]
- Each successor is duplicated:
  - Suboptimal version with \( f\)-value = \( g + w \cdot h \)
  - Optimal version with \( f\)-value = \( w \cdot g + h \)
- Each successor is expanded at most once

**SIPP with FOCAL list (FocalSIPP)**
- Each state can be re-expanded multiple times
- Contains additional list – FOCAL [Pearl and Kim, 1982]
- FOCAL is a subset of OPEN list: \( |n| = \in OPEN, g(n) + h(n) \leq w \cdot f_{\text{max}} \)
- Auxiliary heuristic function \( h_{\text{FOCAL}}(n) \) for choosing a node from FOCAL does not have to be consistent, admissible

An example of instance where SIPP with weighted heuristic (\( w=2 \)) and disallowed re-expansions cannot find a solution. Safe interval in B = [0; 10]

4. Re-expansions Analysis

Re-expanded* states are shown in yellow. The more states corresponding to the grid cell were re-expanded – the darker this cell is (remember that in SIPP numerous states corresponding to the same cell might exist). It is clearly seen that when \( w \) is close 1 the lowest number of re-expansions is achieved by WSIPP,\(_d\) when \( w \) is large – by FocalSIPP, when \( w \) is “medium” – by WSIPP,\(_w\)

5. Experimental evaluation

Maps:
- Empty 64x64
- Warehouse 64x64
- Rooms 64x64
- Den252d 25x256

Action model:
- w (without) rotations
- w (with) rotations

Grid connectivity: 8, 16, 32

250 dynamic obstacles
100 instances per each map

Heuristic functions:
- \( h \) – Euclidean distance
- \( h_{\text{FOCAL}} \) – hops-to-goal

Suboptimality factor: 1.01, 1.05, 1.1, 1.25, 1.5, 1.75, 2, 3, 4, 5

References

