When Perfect is not Good Enough: On the Search Behaviour of Symbolic Heuristic Search

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Symbolic search and heuristic search are two successful approaches to optimal planning.

**Symbolic Planner**
- Symbolic state representation
- Blind search

**Heuristic Planner**
- Explicit state representation
- Informed search
  - Symbolic heuristic representation
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Symbolic Heuristic Planner?
Symbolic Search for Optimal Planning

- Operations on sets of states
- $S \subseteq S$ represented by characteristic function $\chi_S$
- Manipulating $S \Rightarrow$ Transforming $\chi_S$
  - E.g. $S \cap S' \Rightarrow \chi_S \land \chi_{S'}$
- Binary Decision Diagrams (BDDs)
- Search performance depends on the size of BDDs
Symbolic Heuristic Search - BDDA*

Given a set of states $S$, split it according to their $h$-value $S'_i = S \land H_i$. 

$$
\begin{align*}
S \land H_0 & \rightarrow S'_0 \\
S \land H_1 & \rightarrow S'_1 \\
S \land H_2 & \rightarrow S'_2 \\
S \land H_3 & \rightarrow S'_3 \\
\end{align*}
$$

I \rightarrow S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_4
Symbolic Heuristic Search - BDDA*

Given a set of states $S$, split it according to their $h$-value $S' = S \land H_i$. 

![Diagram of state splitting](image)
Symbolic Heuristic Search - BDDA

Given a set of states $S$, split it according to their $h$-value $S'_i = S \land H_i$.

\[
\begin{align*}
S & \land H_0 \\
S' & \quad S'_0 \\
S & \land H_1 \\
S' & \quad S'_1 \\
S & \land H_2 \\
S' & \quad S'_2 \\
S & \land H_3 \\
S' & \quad S'_3
\end{align*}
\]
Given a set of states $S$, split it according to their $h$-value $S'_i = S \land H_i$. 

\[
S \land H_0 \quad S \land H_1 \quad S \land H_2 \quad S \land H_3
\]

\[
S'_0 \quad S'_1 \quad S'_2 \quad S'_3
\]
Given a set of states $S$, split it according to their $h$-value $S'_i = S \land H_i$. 

$S \land H_0$ 

$S' = S \land H_1$ 

$S' = S \land H_2$ 

$S' = S \land H_3$ 

$I_0$ 

$I_1$ 

$I_2$ 

$I_3$ 

$I_4$ 

$g$ 

$h$
Given a set of states $S$, split it according to their $h$-value $S'_i = S \land H_i$. 
Symbolic Heuristic Search - BDDA*

Given a set of states $S$, split it according to their $h$-value $S'_i = S \land H_i$. 

$$S \land H_0 \rightarrow S'_0$$
$$S \land H_1 \rightarrow S'_1$$
$$S \land H_2 \rightarrow S'_2$$
$$S \land H_3 \rightarrow S'_3$$
Consistent heuristics reduce the number of necessary state expansions.

Heuristic computation and state evaluation are expensive [JVB08].

Observation

A BDD $B_{S'}$ can be larger than BDD $B_S$ although the set of states $S'$ is a strict subset of $S$, i.e. $S' \subsetneq S$.

In symbolic search, the search performance is not directly related to the number of explicit states that have to be expanded.
Splitting BDDs according to heuristic values can increase or decrease the sizes of the resulting BDDs.

- In the worst case exponentially
- Even with the perfect heuristic $h^*$
Theoretical Results

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Exponential increase or decrease in search performance!
Empirical Results – BDD Sizes

- BDDA*
- Fraction perfect heuristics
  - $\sim c \cdot h^*$
  - Precomputed
  - Multiple BDDs
- Domains from IPCs
Empirical Results – Runtime

Similar results for symbolic bidirectional search.

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Empirical Results – Runtime

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Conclusion

- Heuristic computation and state evaluation are expensive [JVB08].
- BDDs can become exponentially larger by using heuristics.
  - Even with the perfect heuristics $h^*$
- Overall target: small BDDs
- Fewer States $\iff$ smaller BDDs
- Similar results for uni- and bidirectional search
- $\not\iff$ Heuristics do not pay off in general!