Strengthening Potential Heuristics with Mutexes and Disambiguations

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October 15, 2020

Background

Finite Domain Representation (FDR)

- Multi-valued variables: $\mathcal{V} = \{V_1, V_2, \dots\}$,
- State s is an assignement to variables $\mathcal{V}$,
- Operator $o = \langle \mathrm{pre}(o), \mathrm{eff}(o) \rangle$, $\mathrm{pre}(o)$, $\mathrm{eff}(o)$ partial assignments to $\mathcal{V}$,
- Initial state I,
- Goal specification G is a partial assignment.

Mutex

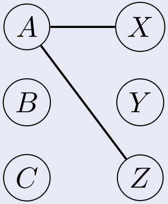
- A set of facts $\mathcal{M}$ such that $\mathcal{M} \not\subseteq s$ for every reachable state s.
- Facts from each variable (mutex group) are pairwise mutex.
- We can infer mutexes with h^m heuristic.

Disambiguation

Disambiguation of a variable V for a partial state p is a set of facts from V consistent with p given a set of mutexes.

Variables:
$\mathrm{dom}(V_1) = \{A, B, C\}$
$\mathrm{dom}(V_2) = \{X, Y, Z\}$

Mutexes:
$\{V_1{=}A, V_2{=}X\}$
$\{V_1{=}A, V_2{=}Z\}$



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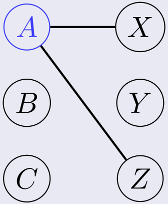

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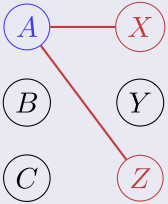

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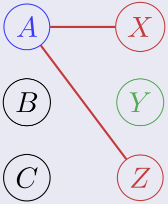

Partial state: $p = \{V_1{=}A\} \longrightarrow \{V_1{=}A, V_2{=}Y\}$

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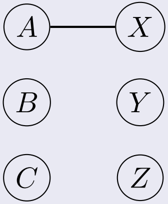

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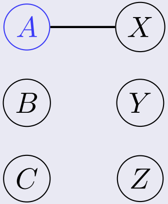

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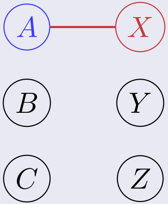

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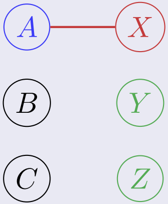

Partial state: $p = \{V_1{=}A\} \longrightarrow \{V_1{=}A, (V_2{=}Y \text{ or } V_2{=}Z)\}$

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Disambiguation of a variable V for a partial state p is a set of facts from V consistent with p given a set of mutexes.

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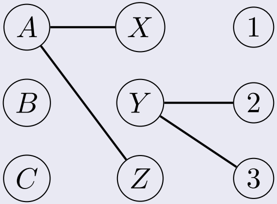

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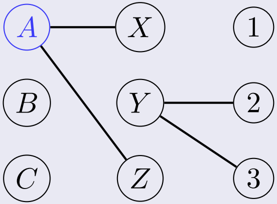

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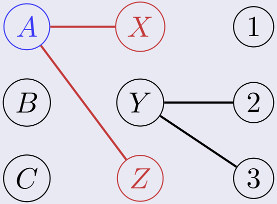

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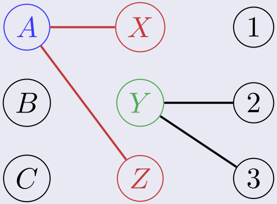

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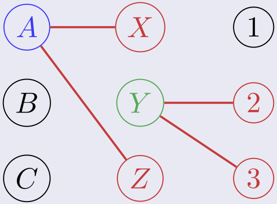

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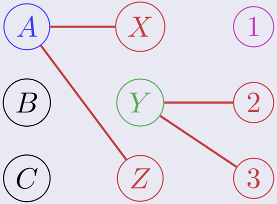

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Potential Heuristics

A **potential function** is a function $\mathrm{P} : \mathcal{F} \mapsto \mathbb{R}$.
A **potential heuristic** for P maps each reachable state s to the sum of potentials of facts in s, i.e., $h^{\mathrm{P}}(s) = \sum_{f \in s} \mathrm{P}(f)$.

Goal awarness

For every goal state s_G:

$$\sum_{f \in s_G} \mathrm{P}(f) \leq 0$$

Consistency

For every operator o and every reachable state s where o is applicable:

$$\sum_{f \in s} \mathrm{P}(f) - \sum_{f \in o[\![s]\!]} \mathrm{P}(f) \leq \mathrm{c}(o)$$

Disambiguation of Goal Specification

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Goal specification: $G = \{V_1{=}A\}$

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$$\mathtt{P}(V_1{=}A) + \max_{v \in \mathrm{dom}(V_2)} \mathtt{P}(V_2{=}v) + \max_{v \in \mathrm{dom}(V_3)} \mathtt{P}(V_3{=}v) \leq 0$$

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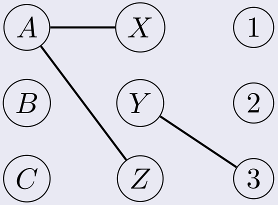

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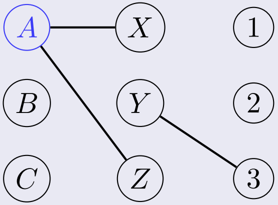

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$$\mathrm{P}(V_1{=}A) + \mathrm{P}(V_2{=}Y) + \max(\mathrm{P}(V_3{=}1), \mathrm{P}(V_3{=}2)) \leq 0$$

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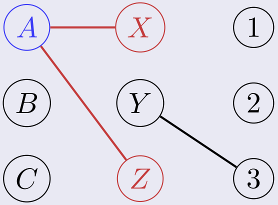

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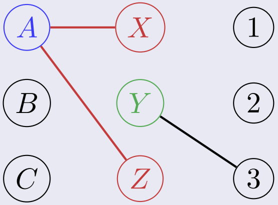

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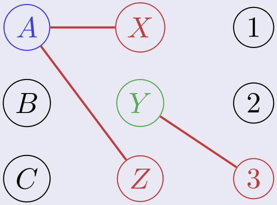

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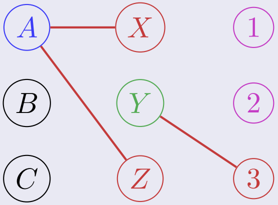

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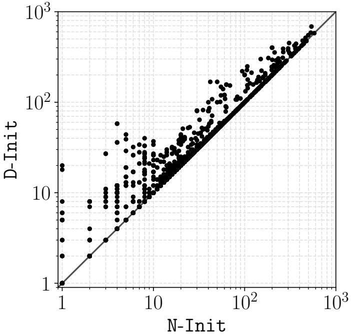
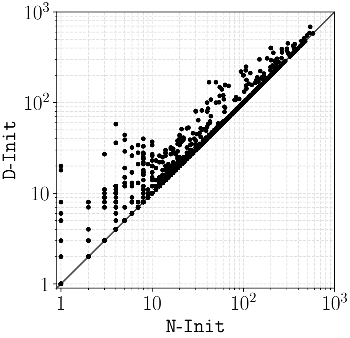
Potential Heuristics with Disambiguation



	Init		All		$\mathtt{Div}_{1k}$		$\hat{\mathtt{S}}_{1k}$	
	N	D	N	D	N	D	N	D
Coverage	921	938	903	927	932	957	937	961

Potential Heuristics with Disambiguation



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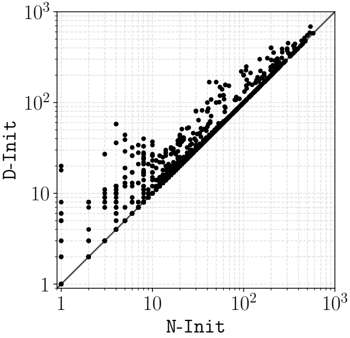

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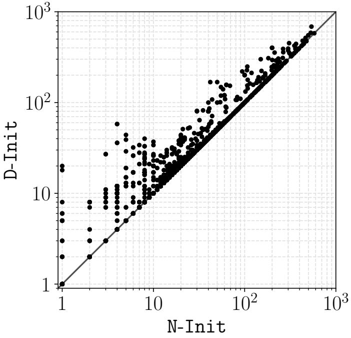

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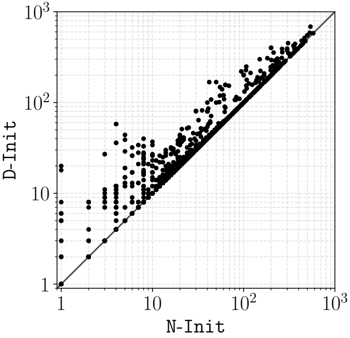

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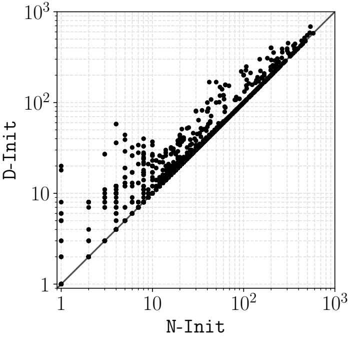

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Constraint on Initial State

Enforcing heuristic value for a state

We can always enforce a minimal heuristic value $h(s)$ for a specific state s by the constraint

$$\sum_{f \in s} \mathrm{P}(f) \geq h(s)$$

Therefore, we can first compute the potential heuristic h_I^{P} for the initial state and then enforce its value for other objective function by the constraint:

$$\sum_{f \in I} \mathrm{P}(f) \geq h_I^{\mathrm{P}}(I)$$

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Enforcing heuristic value for a state

We can always enforce a minimal heuristic value $h(s)$ for a specific state s by the constraint

$$\sum_{f \in s} \mathtt{P}(f) \geq h(s)$$

Therefore, we can first compute the potential heuristic $h_I^{\mathtt{P}}$ for the initial state and then enforce its value for other objective function by the constraint:

$$\sum_{f \in I} \mathtt{P}(f) \geq h_I^{\mathtt{P}}(I)$$

	All		All+I		$\hat{\mathtt{S}}_{1k}$		$\hat{\mathtt{S}}_{1k}$+I		lmc	ms
	N	D	N	D	N	D	N	D		
Coverage	903	927	964	1 001	937	961	949	985	911	895

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Summary

- Disambiguation with mutexes is relatively cheap way to improve potential heuristics.
- Mutexes can be used to a more accurate estimation of the number of reachable states (details in the paper).
- Adding constraint for the initial state with the maximization of objective function describing average state brings best of both methods.