Strengthening Potential Heuristics with Mutexes and Disambiguations

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Background

Finite Domain Representation (FDR)

- Multi-valued variables: $\mathcal{V} = \{V_1, V_2, \ldots\}$,
- State $s$ is an assignment to variables $\mathcal{V}$,
- Operator $o = \langle \text{pre}(o), \text{eff}(o) \rangle$, pre($o$), eff($o$) partial assignments to $\mathcal{V}$,
- Initial state $I$,
- Goal specification $G$ is a partial assignment.

 Mutex

- A set of facts $\mathcal{M}$ such that $\mathcal{M} \not\subseteq s$ for every reachable state $s$.
- Facts from each variable (mutex group) are pairwise mutex.
- We can infer mutexes with $h^m$ heuristic.
Disambiguation of a variable $V$ for a partial state $p$ is a set of facts from $V$ consistent with $p$ given a set of mutexes.

**Variables:**
- $\text{dom}(V_1) = \{A, B, C\}$
- $\text{dom}(V_2) = \{X, Y, Z\}$

**Mutexes:**
- $\{V_1=A, V_2=X\}$
- $\{V_1=A, V_2=Z\}$

**Partial state:** $p = \{V_1=A\}$
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Partial state: $p = \{V_1=A\} \rightarrow \{V_1=A, V_2=Y\}$
Disambiguation of a variable $V$ for a partial state $p$ is a set of facts from $V$ consistent with $p$ given a set of mutexes.

Variables:
$\text{dom}(V_1) = \{A, B, C\}$
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Partial state: $p = \{V_1=A\}$
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Mutexes:
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- $V_1 = B, V_2 = Y$
- $V_1 = C, V_2 = Z$

Partial state: $p = \{V_1 = A\}$
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**Partial state:**
- $p = \{V_1=A\}$
- $\rightarrow \{V_1=A, (V_2=Y \text{ or } V_2=Z)\}$
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**Partial state:** $p = \{V_1=A\}$

![Diagram](image-url)
Disambiguation

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- $\{V_2=Y, V_3=3\}$

**Partial state:** $p = \{V_1=A\} \rightarrow \{V_1=A, V_2=Y, V_3=1\}$
A potential function is a function $P : \mathcal{F} \mapsto \mathbb{R}$.

A potential heuristic for $P$ maps each reachable state $s$ to the sum of potentials of facts in $s$, i.e., $h^P(s) = \sum_{f \in s} P(f)$.

**Goal awareness**
For every goal state $s_G$:

$$\sum_{f \in s_G} P(f) \leq 0$$

**Consistency**
For every operator $o$ and every reachable state $s$ where $o$ is applicable:

$$\sum_{f \in s} P(f) - \sum_{f \in o[s]} P(f) \leq c(o)$$
Disambiguation of Goal Specification

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Goal specification: $G = \{V_1=A\}$
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Goal specification: $G = \{V_1=A\}$

$$P(V_1=A) + \max_{v \in \text{dom}(V_2)} P(V_2=v) + \max_{v \in \text{dom}(V_3)} P(V_3=v) \leq 0$$
Disambiguation of Goal Specification

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Goal specification: \( G = \{ V_1 = A \} \)

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\end{align*}
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P(V_1=A) + P(V_2=Y) + \max(P(V_3=1), P(V_3=2)) \leq 0
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$P(V_1=A) + P(V_2=Y) + \max(P(V_3=1), P(V_3=2)) \leq 0$
Potential Heuristics with Disambiguation

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Potential Heuristics with Disambiguation

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N-Init
1
10
10^2
10^3
D-Init
```

![Graph showing the relationship between D-Init and N-Init](image)

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Coverage

October 15, 2020
Potential Heuristics with Disambiguation

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Potential Heuristics with Disambiguation

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Potential Heuristics with Disambiguation

![Graph showing the relationship between N-Init and D-Init coverage](image)

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Enforcing heuristic value for a state

We can always enforce a minimal heuristic value $h(s)$ for a specific state $s$ by the constraint

$$\sum_{f \in s} P(f) \geq h(s)$$

Therefore, we can first compute the potential heuristic $h^P_I$ for the initial state and then enforce its value for other objective function by the constraint:

$$\sum_{f \in I} P(f) \geq h^P_I(I)$$
**Constraint on Initial State**

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Therefore, we can first compute the potential heuristic $h^P_I$ for the initial state and then enforce its value for other objective function by the constraint:

$$\sum_{f \in I} P(f) \geq h^P_I(I)$$

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Enforcing heuristic value for a state

We can always enforce a minimal heuristic value $h(s)$ for a specific state $s$ by the constraint

$$\sum_{f \in s} P(f) \geq h(s)$$

Therefore, we can first compute the potential heuristic $h^P_I$ for the initial state and then enforce its value for other objective function by the constraint:

$$\sum_{f \in I} P(f) \geq h^P_I(I)$$

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**Constraint on Initial State**

Strengthening Potential Heuristics with Mutexes and Disambiguations

October 15, 2020 8 / 9
Disambiguation with mutexes is relatively cheap way to improve potential heuristics.

Mutexes can be used to a more accurate estimation of the number of reachable states (details in the paper).

Adding constraint for the initial state with the maximization of objective function describing average state brings best of both methods.