On the Computational Complexity of Multi-Agent Pathfinding on Directed Graphs

Albert-Ludwigs-Universität Freiburg

Bernhard Nebel

University of Freiburg, Germany

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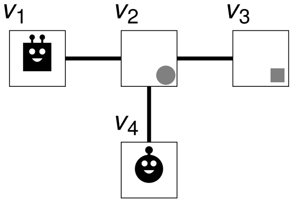
Multi-Agent Path Finding

- **Given**: A set of agents A, an undirected, simple graph $G = (V, E)$, an initial state modelled by an injective function $\alpha_0 : A \rightarrow V$ and a goal state modelled by another injective function $\alpha_* : A \rightarrow V$.
- **Question**: Can α_0 be transformed into α_* by movements of single agents without collisions?



- Can we find a plan to move the square agent S to v_3 and the circle agent C to v_2?
- Yes, we can!

Computational properties of MAPF

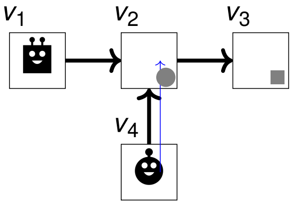
- Deciding MAPF plan existence can be solved in $O(n^3)$ time and the plan length can be bounded by $O(n^3)$ movement actions (Kornhauser, Miller & Spirakis 84).
- Finding a shortest plan is NP-hard (Goldreich 84, Ratner & Warmuth 86).
- A number of variations of the problem (e.g. parallel movements) have been studied and a number of algorithms have been devised since then.
- Open problem since 1984: What is the computational complexity of MAPF on directed graphs (*diMAPF*)?

Special case of diMAPF

- diMAPF solvability is a polynomial problem on strongly bi-connected directed graphs, provided there are at least 2 unoccupied nodes (Botea, Bonusi & Surynek 18, Botea, & Surynek 15).
- The authors plan to extend their analysis to other classes of digraphs.

Complications with DAGs

- In undirected graphs and strongly bi-connected graphs, one can sort of restore a partial state after some agents have been moved to some other places.
- There are no dead ends for solvable instances.
- In DAGs you can easily fail!



- Order matters! You need to look into the future.

A lower bound for diMAPF

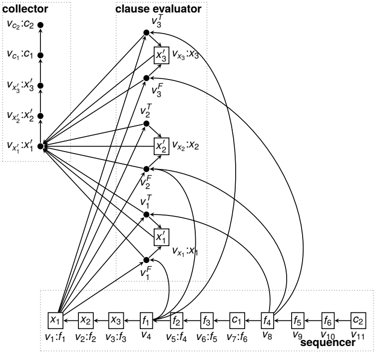
Theorem

diMAPF solvability is NP-hard.

Proof sketch.

Reduction from 3SAT.

Example reduction for $(x_1 \lor x_2 \lor \neg x_3) \land (\neg x_1 \lor x_2 \lor x_3)$:



An upper bound for diMAPF

- On DAGs, only polynomially many moves in each plan are possible, i.e., diMAPF on DAGs is NP-complete.
- On digraphs with cycles, however, one could generate exponentially many different configurations, so membership in NP is not obvious.

Theorem

diMAPF solvability on general directed graphs is in PSPACE.

Proof sketch.

diMAPF solvability can be understood as a special case of plan existence for propositional STRIPS. □

A conditional upper bound for diMAPF

Cycles in digraphs are the culprits that stop us from proving membership in NP.

Hypothesis (Short solution hypothesis)

Solution length for diMAPF on strongly connected digraphs is polynomial.

Theorem

diMAPF solvability is NP-complete, provided the short solution hypothesis is true.

Proof sketch.

Each agent can only enter a strongly connected component once, and leave it once. So if there are short solutions for all strongly connected components, there will be one for the overall graph. □

Conclusion

- Identified problem that is open since more than 35 years (but did anybody notice that it was open?)
- Demonstrated that a Kornhauser-style algorithm for directed graphs is impossible.
- Results generalize to variations with parallel moves.
- Open problem: Is the short solution hypothesis true?