

# Hierarchical Graph Traversal for Aggregate $k$ Nearest Neighbors Search in Road Networks

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ICAPS 2020 SEMINAR

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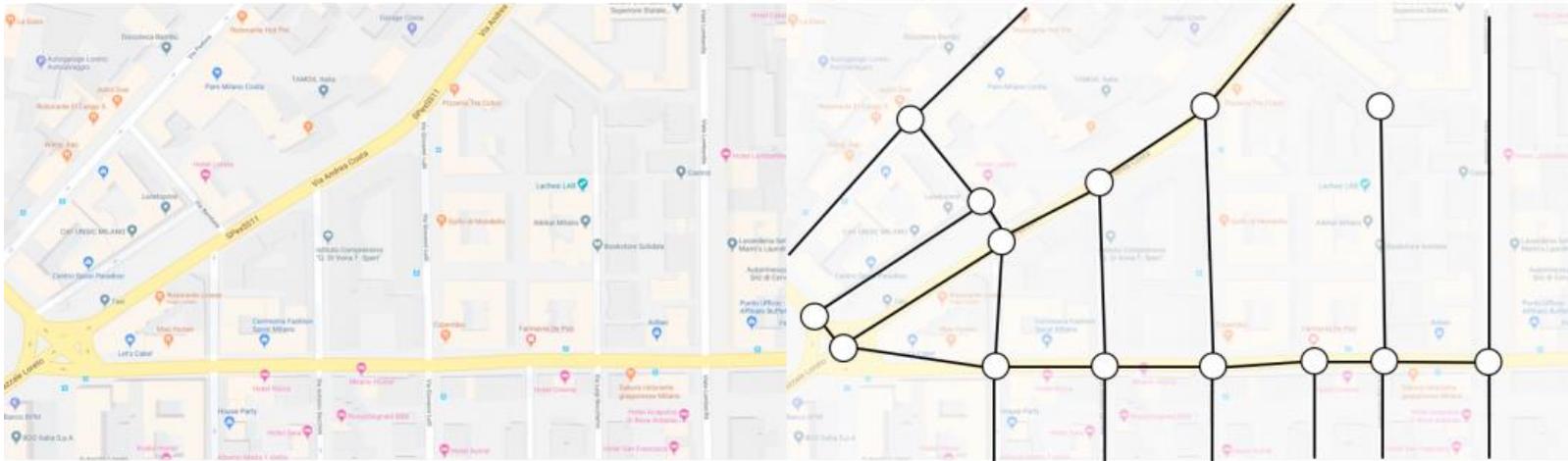
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# Background: Road Network Graph

- Input: Road network graph  $G = (V, E)$
- Vertex set  $V$ : Road intersections
- Edge set  $E$ : Road segments
- Each edge has weight: e.g. travel time

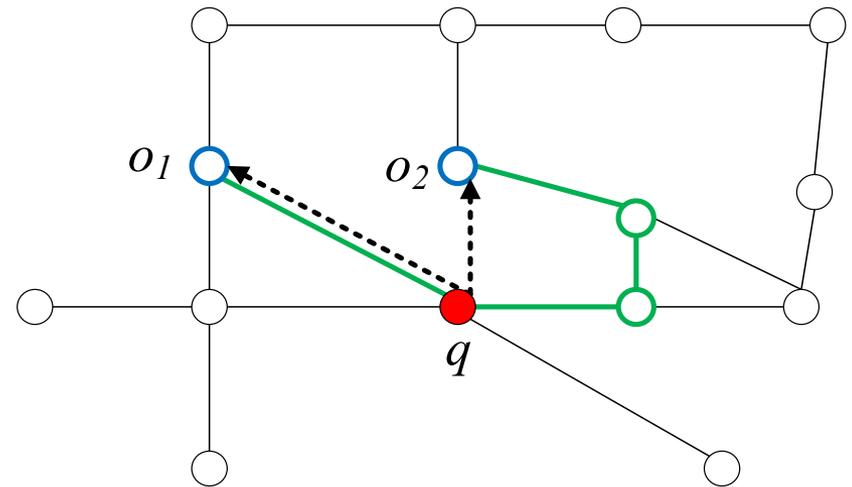
Source:

<https://magazine.impactscool.com/en/speciali/google-maps-e-la-teoria-dei-grafi/>



# Background: k Nearest Neighbour (kNN) Queries

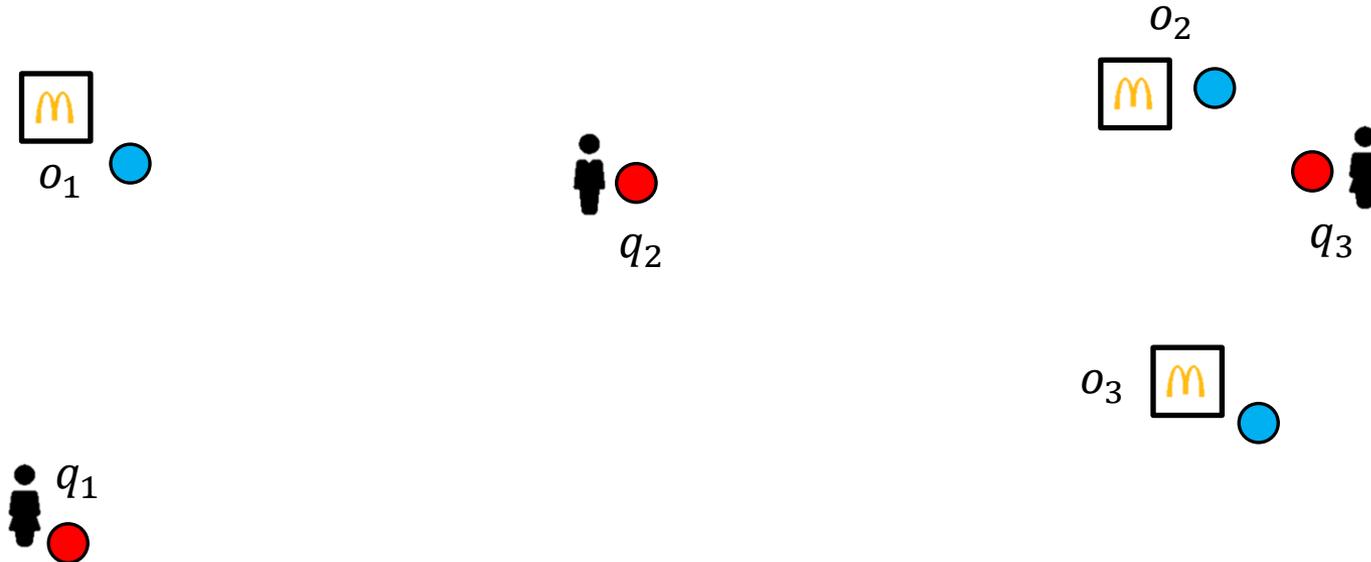
- Input: Object set  $O \subseteq V$  (e.g. all restaurants)
- Input: Agent location  $q \in V$  (e.g. a diner)
- kNN Query: What is the nearest object to  $q$ ?
  - *By Euclidean Distance:  $o_2$*
  - *By Network Distance:  $o_1$*
- More accurate + versatile



# Our Problem: Aggregate $k$ Nearest Neighbours (AkNN)

- AkNN: Find the nearest object to **multiple** agents
  - Example: Three friends (agents) want to meet at a McDonalds (objects). Which object to meet at?

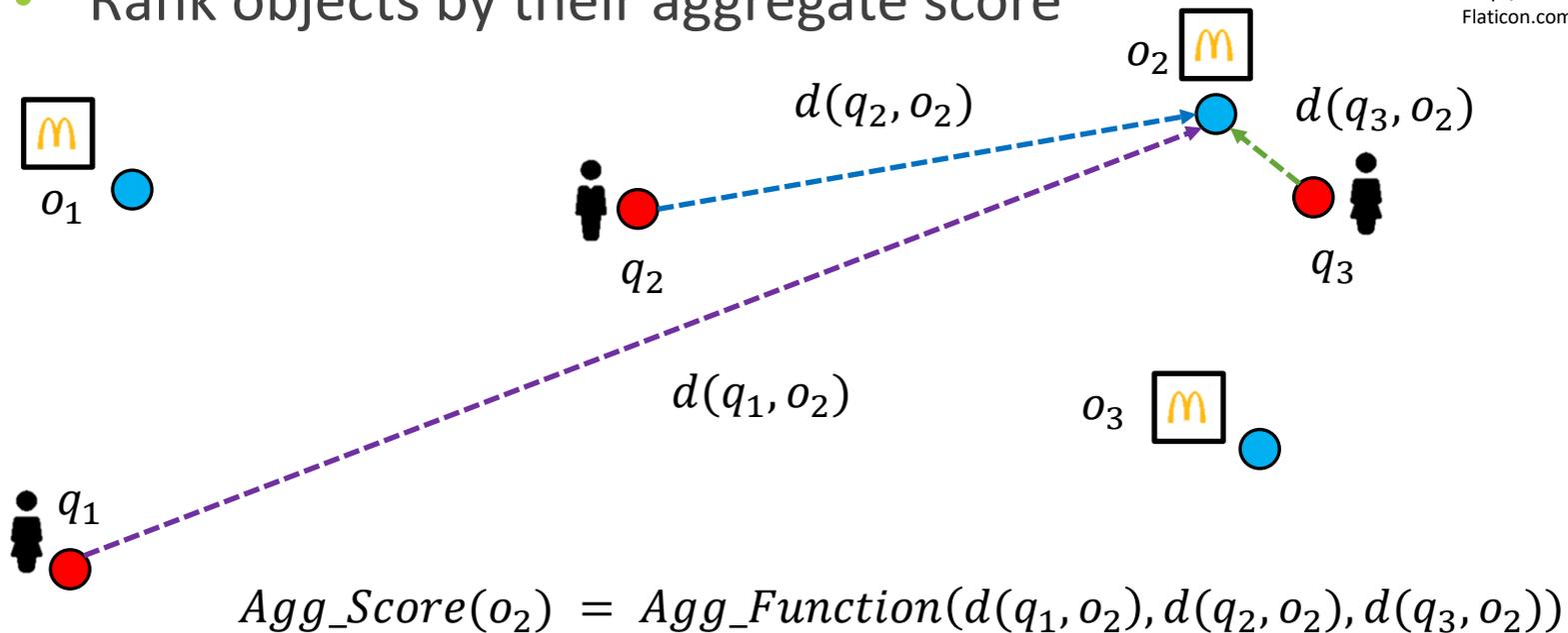
Sources: Google Maps, McDonalds, Flaticon.com



# Our Problem: AkNN

- Input: Aggregate Function (e.g. SUM), Agent Set  $Q \subseteq V$
- Aggregate individual distances from each agent
- Rank objects by their aggregate score

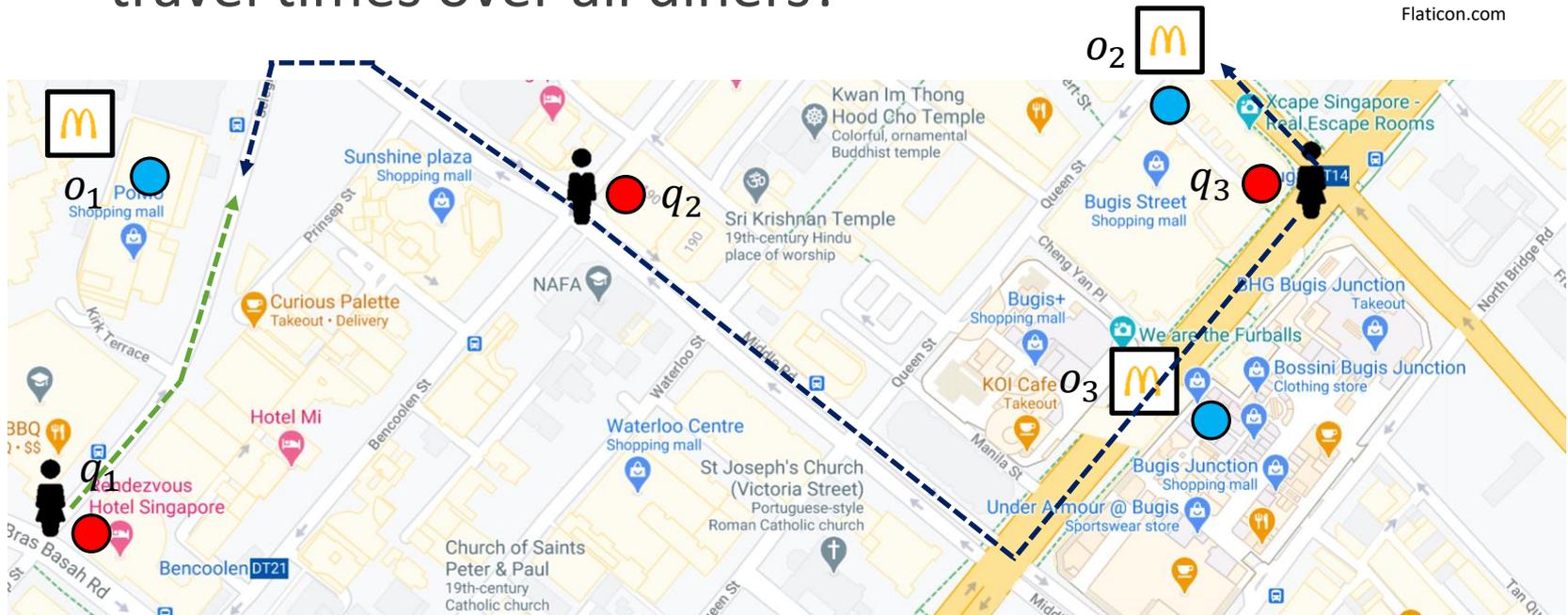
Sources: Google Maps, McDonalds, Flaticon.com



# Our Problem: AkNN

- Still using network distance for accuracy/versatility
- Example: Which McDonalds minimises the SUM of travel times over all diners?

Sources: Google Maps, McDonalds, Flaticon.com



# Motivation

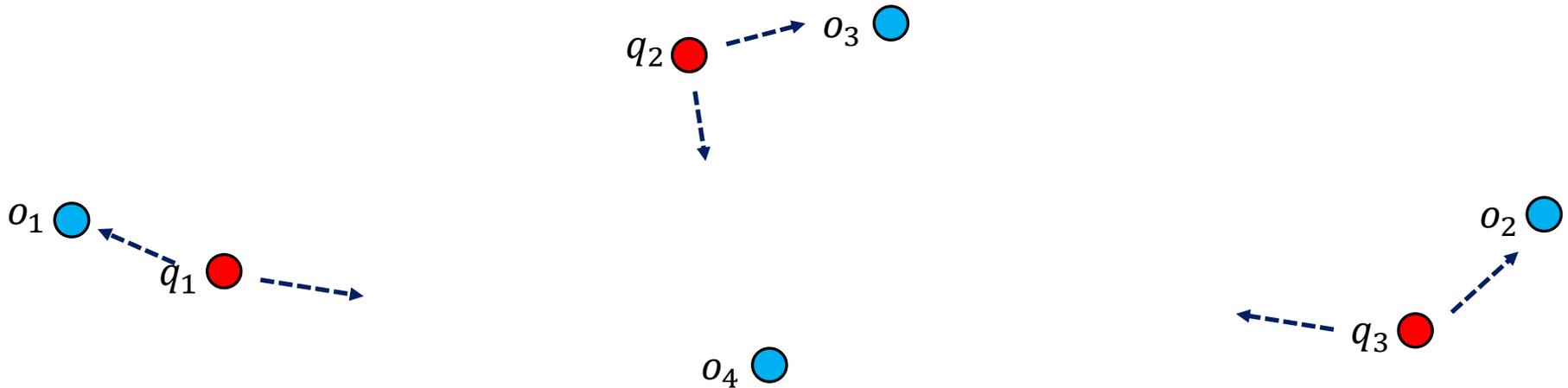
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- Inefficient to compute distance to every object
- Typical Solution: heuristically retrieve likely candidates until all results found
- But existing heuristics are either:
  - (a) borrowed from kNN => not suitable for AkNN
  - (b) not accurate enough for network distance

# Expansion Heuristics

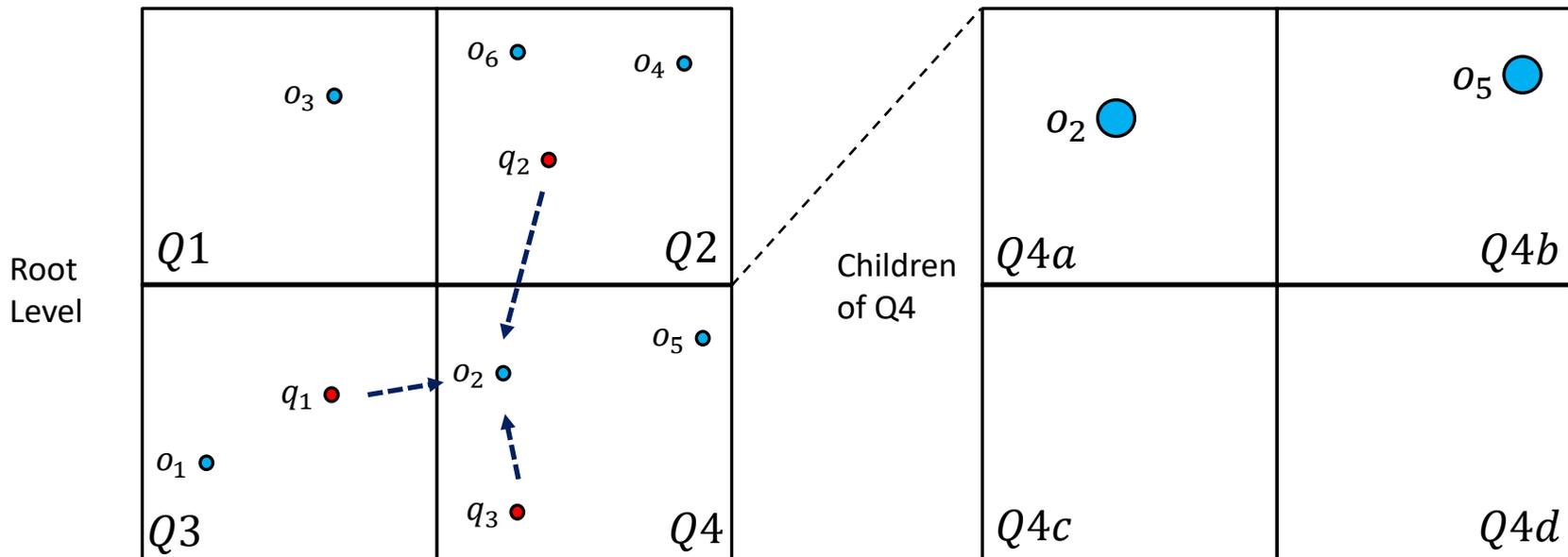
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- Borrowed from kNN search heuristics: expand from each query vertex
- But best AkNN candidates unlikely to be near any one query vertex



# Hierarchical Search Heuristic

- Divide space to group objects => recursively
- Search “promising” regions top-down (recursively)
- Pinpoint best candidate anywhere in space

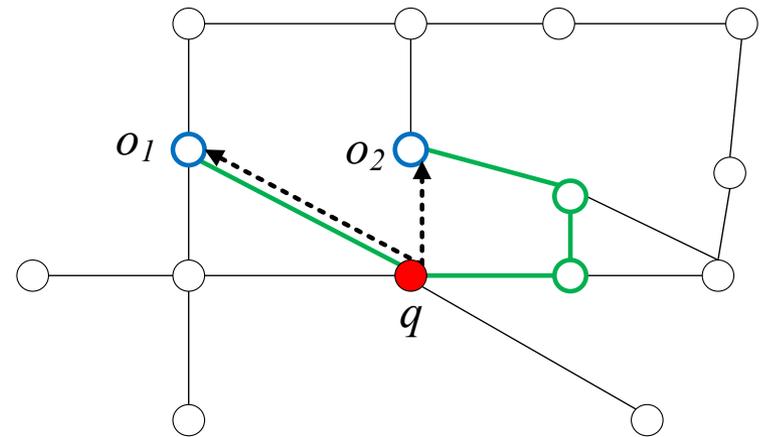


# Hierarchical Search

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- How do we decide which regions are “promising”?
- Use **lower-bound score** for all objects in a region
- Past Work: R-tree + Euclidean distance lower-bound
- Not accurate for road network distance

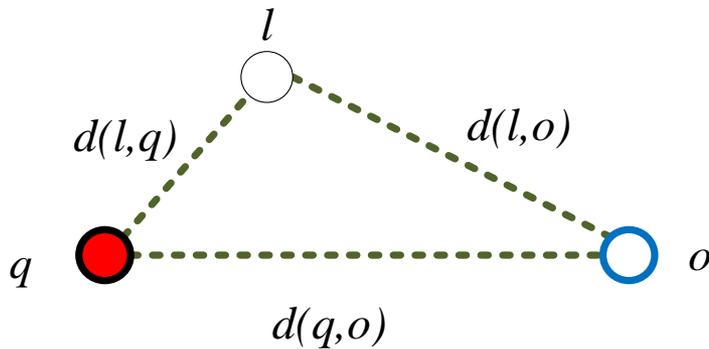
Data structure needed for **accurate** hierarchical lower-bound search in graphs



# Landmark Lower-Bounds

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- Pre-compute distances from *landmark* vertices
- Use triangle inequality to compute lower-bound
- Only allows small numbers of landmarks (space cost)
- Not suitable for hierarchical search

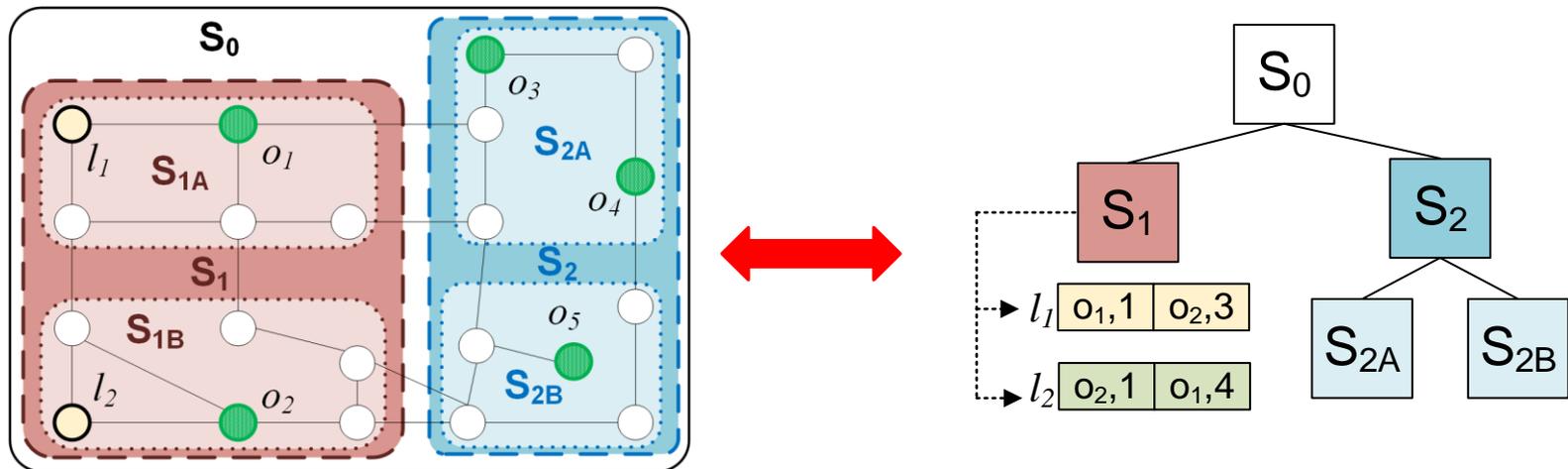


$$d(q, o) \leq |d(l, o) - d(l, q)|$$

*Choose tightest LLB over a set of multiple landmarks*

# Compacted-Object Landmark Tree (COLT) Index

- Partition graph recursively => subgraph tree
- Choose localised landmarks in every subgraph
- Compact based on object set  $O$



# COLT

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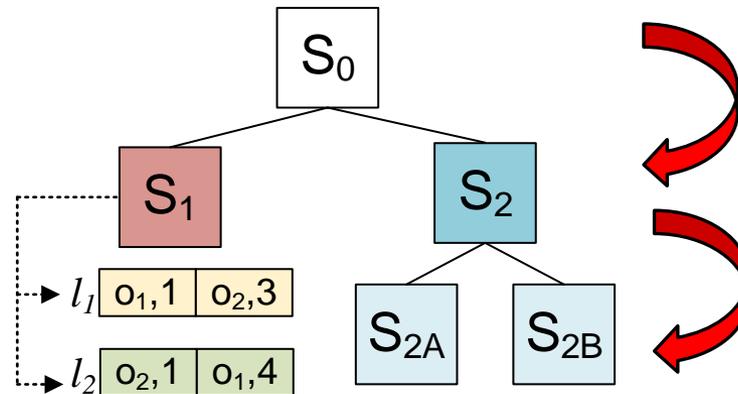
- Non-leaf + leaf nodes stores
  - $M^-$ : min distance to any object in subgraph from landmark
  - $M^+$ : max distance to any object in subgraph from landmark
- Enables accurate lower-bound for any tree node

$$LB_{l_i}(n_C, q) = \begin{cases} d(l_i, q) - M^+ & \text{if } d(l_i, q) \geq M^+ \\ M^- - d(l_i, q) & \text{if } d(l_i, q) \leq M^- \\ 0 & \text{else} \end{cases}$$

# Hierarchical Traversal in COLT

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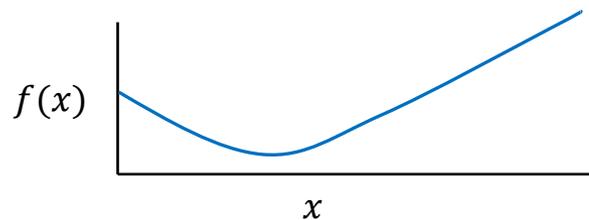
- Top-down search from root node
- Compute lower-bound for child using equation
- Recursively evaluate child with best score



# Hierarchical Traversal in COLT

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- Leaf nodes store Object Distance List
- Find object with minimum aggregate lower-bound
- Interestingly common functions preserve convexity!
- Easily found using modified binary search



Object	$o_4$	$o_2$	$o_5$	$o_1$	$o_3$
Distance	2	5	7	8	12

# Experimental Setup

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- Dataset: US Road Network Graph from DIMACS
  - $|V| = 23,947,347$  vertices,  $|E| = 57,708,624$  edges
  - Real-World POIs from OSM for US
- Comparison against IER and NVD
  - IER: hierarchical search using Euclidean heuristic
  - NVD: state-of-the-art expansion heuristic

# Query Time: Real-World POIs

- COLT up to an order of magnitude faster!
- COLT performs better on dense POI sets
- Heuristics is less important on sparse POI sets

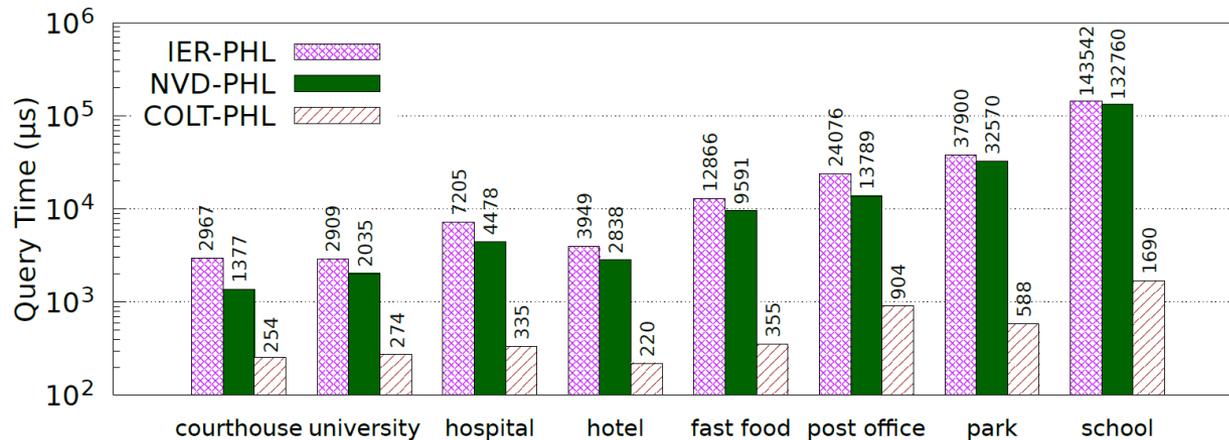
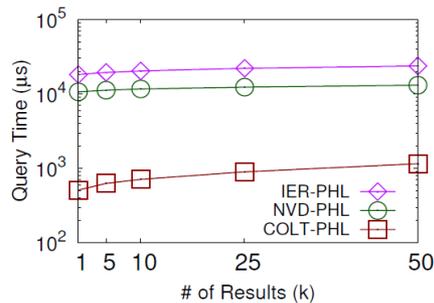


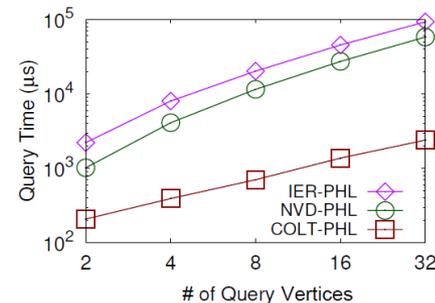
Figure 4: Performance on different real-world POI sets

# Sensitivity Analysis

- COLT maintains improvement for
  - Varying parameters ( $k$ , number of agents)
  - Varying aggregate functions (MAX, SUM)
  - Heuristic efficiency metrics
- Comes at a lightweight pre-processing cost



(b) Varying  $k$



(c) Varying  $|Q|$

Figure 5: Performance for *max* function

Thank You!

Questions?