Dynamic Controllability and (J,K)-Resiliency in Generalized Constraint Networks with Uncertainty

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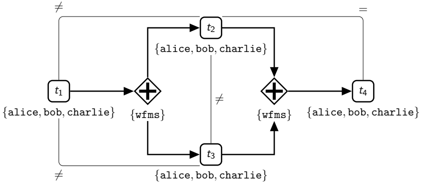
Outline and Contribution

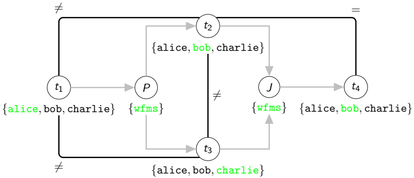
1. Background on Constraint Networks (CNs)
2. Generalized CNs with Uncertainty
3. Dynamic Controllability and (J,K)-Resiliency
4. Complexity Analysis
5. Conclusions and Future Work

Constraint Networks for Resource Allocation

Process with Resources



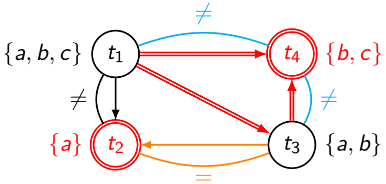
Partially ordered CN



How to get a plan:

1. Compute a topological sort (polynomial-time step)
2. Solve the constraint network (NP-hard step)

Generalized Constraint Networks with Uncertainty

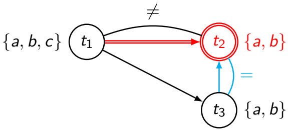


1. we execute (=pick *then* assign values) to variables one at a time.
2. t_2 and t_4 have uncontrollable value assignments.
3. t_3 and t_4 have uncontrollable pickings (=once active their execution may arrive "anytime").
4. $(t_1 < t_2) \wedge (t_1 \neq t_2) \wedge (t_1 \neq t_4 \vee t_3 \neq t_4) \wedge$ $(t_3 < t_2 \vee t_2 = t_3)$.

- Variables with controllable and uncontrollable value assignment (double circles)
- Variables with uncontrollable picking and (acyclic) activation constraints (double arrows)
- Constraints language: arbitrary boolean formula over relational and partial order constraints.
- Allow for defining problems as 2-player games Controller-Nature.

(qualitative time approach)

Dynamic Controllability



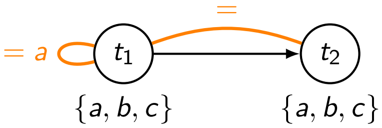
Dynamically Controllable.

Controller's Winning Strategy

- Controller picks t_1 and assigns c to it (t_2 is now ready for picking)
- If Nature doesn't pick t_2,
  - Controller picks t_3 and assigns any value to it.
  - Nature picks t_2 and assigns a value to it
- If Nature picks t_2, she also assigns a value to it.
- Controller picks t_3 and assigns to it the same value assigned to t_2

- Round-based Game between Controller and Nature.
- At every round a variable is *first* picked, *then* assigned a value.
- Both Controller and Nature can pick and/or assign (=4 possible cases).
- Nature has priority over Controller when picking.
- Dynamic controllability = Winning strategy for Controller.
- Uncontrollability = Winning strategy for Nature.

(J,K)-Decremental Resiliency



Not (2,2)-decrementally resilient.

Nature's Winning Strategy

- Nature removes a at Round 1
- Controller picks t_1 and assigns b or c to it at Round 1.
- Nature removes the value that Controller assigned to t_2 at Round 2.
- Controller cannot assign the same value to t_2 at Round 2 (i.e., loses the game).

- On top of dynamic controllability.
- J and K are natural numbers
- For maximum J rounds Nature *strikes* by removing overall up to K values
- After that, Controller and Nature pick and assign values to variables (among those remained).
- We still look for a winning strategy

J depends on K: $J \leq K$.

(J,K)-Dynamic Resiliency

$(1, 1)$-**dynamically resilient**

Controller's Winning Strategy

- If Nature removes a at round 1,
  - Controller picks t_2 and assigns b to it.
  - Controller picks t_1 and assigns a to it.
- If Nature doesn't remove a at Round 1.
  - Controller picks t_1 and assigns a to it.
  - Controller picks t_2 and assigns any value to it among those available.

- Like decremental, but with value re-entering at the end of round.
- For maximum J times, at the start of every round, Nature *strikes* by removing up to K values.
- After that, Controller and Nature pick and assign values to variables (among those remained).
- Before the next round begins, all removed values become available again.

J is independent from K

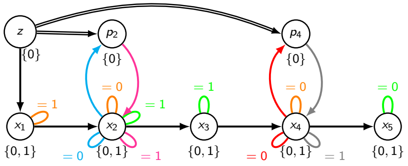
PSPACE-completeness

Theorems

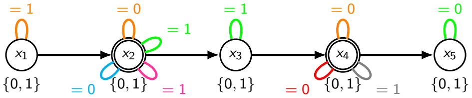
1. **Dynamic controllability of GCNUs is PSPACE-complete.**
   - **Hardness:** reduction from true quantified boolean formula.
   - **Membership:** AND/OR search tree with polynomially bounded depth.
2. **(J,K)-Decremental and (J,K)-Dynamic Resiliency of GCNUs are PSPACE-complete.**
   - **Hardness:** Inherited from dynamic controllability
   - **Membership:** Again, AND/OR search trees.

$$\exists x_1 \forall x_2 \exists x_3 \forall x_4 \exists x_5 (x_1 \vee \neg x_2 \vee \neg x_4) \wedge (x_2 \vee x_3 \vee \neg x_5)$$

Uncontrollable pickings only



Uncontrollable variable assignments only



Conclusions and Future Work

Achieved results

- We provided Generalized Constraint Networks with Uncertainty
- We defined Dynamic Controllability, (J,K)-Decremental and (J,K)-Dynamic Resiliency as two-player games
- We classified the complexity of these decision problems

What next?

- Strategy synthesis algorithms for the corresponding search problems